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Numerical Simulation of Magnetized Blood Flow in Cylindrical Vessel via Caputo-Fabrizio Time-Fractional Models

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Abstract

The mathematical modelling of blood flow is a fundamental aspect of bio-fluid mechanics, offering a critical insight into human circulatory system and the development of targeted medical therapies. The magnetohydrodynamic flow of blood containing magnetic particles in a cylindrical artery is investigated using the Caputo–Fabrizio fractional derivative. Fractional order α alters viscoelastic memory, and magnetic field strength (Ha) reduces flow velocity quadratically, according to Laplace-Hankel transform solutions. In contrast to classical models, the results demonstrate close particle-blood contact and validate the non-singular fractional approach. This work provides a critical mathematical foundation for improving magnetic drug delivery systems, enabling precise prediction of therapeutic particle deposition in targeted cancer treatments, and advances customized hemodynamic interventions.

1. Introduction

Blood flow dynamics are crucial for both treating disease and preserving physiological health. Magnetic nanoparticles (MNPs), such Fe_3O_4 particles, are used in the emerging field of magnetic drug targeting to deliver therapeutic medications to specific regions using external magnetic fields. This technique improves treatment precision while lowering systemic side effects. Optimizing such systems, however, requires accurate mathematical models that can capture the complex behaviour of blood flow under magnetic influence [1][2][3].

Conventional blood flow models based on integer-order calculus are unable to adequately capture the viscoelastic memory effects found in blood, a non-Newtonian fluid. Therefore, fractional derivatives have been employed in fractional calculus to represent these memory features. Fractional calculus was initially studied by [4]. Models could now recognize effects present in physical systems with memory by laying the foundation for fractional calculus. The behaviour of viscoelastic fluids and their relationship to biological events might then be sufficiently described by fractional models [5] by addressing both the singularity and numerical variable instability, [6] improved upon earlier models by creating models based on a non-singular exponential kernel.

[6] showed that Laplace transform functions well with their non-singular exponential kernel, enabling the transformation of differential equations into more straightforward algebraic forms in the frequency domain. This approach is especially beneficial for modelling variable blood flow, where the temporal history of the system needs to be faithfully depicted. [7] and [8] utilized this dual-transform technique Laplace for time and Hankel for space to address two-phase fractional models of blood flow with magnetic particles. Once the system has been transformed, solutions are derived in the altered domain, followed by the application of inverse Laplace and Hankel transforms to retrieve physical properties like fluid and particle velocities. This integrated analytical

method allows for the development of semi- analytical or closed-form solutions, which would be unmanageable otherwise due to the complexities introduced by fractional-order terms and magnetic field interactions.

The governing equations are solved analytically using Laplace and finite Hankel transformations. We study the impact of fractional order, magnetic field intensity (Hartmann number), and particle concentration on flow velocities. The findings validate the application of the Caputo–Fabrizio derivative in hemodynamic modelling by showing significant agreement between the novel fractional model (NFDt) and the classical fractional approach (UFDt). This work offers a physiologically realistic and robust framework for developing magnetic drug delivery devices, with great promise for better cardiovascular and cancer treatments [9][10].

2. Methodology

The approach and analytical tools for solving the derivatives problem and simulating the Caputo-Fabrizio Time-Fractional Derivative under a transverse magnetized flow in an asymmetrical cylinder vessel were presented in this part.

2.1 Caputo-Fabrizio Time-Fractional Derivative

The Caputo-Fabrizio time-fractional derivative is a modern fractional operator designed to overcome the limitations of classical fractional derivatives. This is especially important in systems where memory effects are significant, and mathematical singularities create problems for stability and physical realism. It uses an exponential decay kernel instead of a singular power-law, which allows for a smooth and continuous memory response over time. Unlike the Caputo or Riemann-Liouville derivatives, which often have singularities at the lower limit of integration and complicate numerical simulations, the Caputo-Fabrizio derivative maintains finite and stable behavior throughout the domain. This makes it ideal for modelling real-world systems, particularly biological processes like blood flow, where physical responses are non-instantaneous and memory-dependent, yet still smooth and bounded. In this research, the Caputo-Fabrizio derivative reformulates the classical momentum equations that govern blood and particle motion into a fractional-order framework. This approach allows the system to reflect not only the current state of flow but also its dependence on past states, which is crucial for capturing the viscoelastic nature of blood. It enables the model to consider gradual responses to applied magnetic fields and pulsatile pressure gradients, rather than assuming an immediate reaction as integer-order models do.

2.2 Blood flows in Magnetic Fields

The study of blood flow under magnetic fields, widely known as biomagnetic fluid (BFD), explores how magnetic fields influence the movement of fluids in our body particularly blood. It has become an important research area due to its applications in biomedical engineering and clinical therapies. Blood, in which widely known as a conducting fluid due to its ionic contents, responding to magnetic fields, in which allowing the non-invasive methods to influence and monitor the circulatory behavior.

In 2003, [1] laid the groundwork for understanding the biomedical applications of magnetic nanoparticles, in which demonstrates their capacity to be maneuvered through the use of external magnetic fields. [9] further highlighting the potential of magnetic nanoparticles (MNPs) in drug delivery and tumour targeting, showcasing how the relationship between blood flow and magnetic control can be altered to improve the clinical results.

In the studies made by [10], demonstrated and illustrated the potential of magnetic fields in treating haemodynamic diseases such as hypertension and atherosclerosis. Their studies proved that by applying an external magnetic field could significantly alter the flow characteristics of blood, which proving that there are more to explore for the design of therapeutic devices and drug delivery systems. It is also important to note that the investigation of blood flow influenced by magnetic field have piqued the interest of many researchers during the recent years, particularly for its relevance in biomedical fields like magnetic resonance imaging (MRI), targeted drug delivery, and hyperthermia treatments.

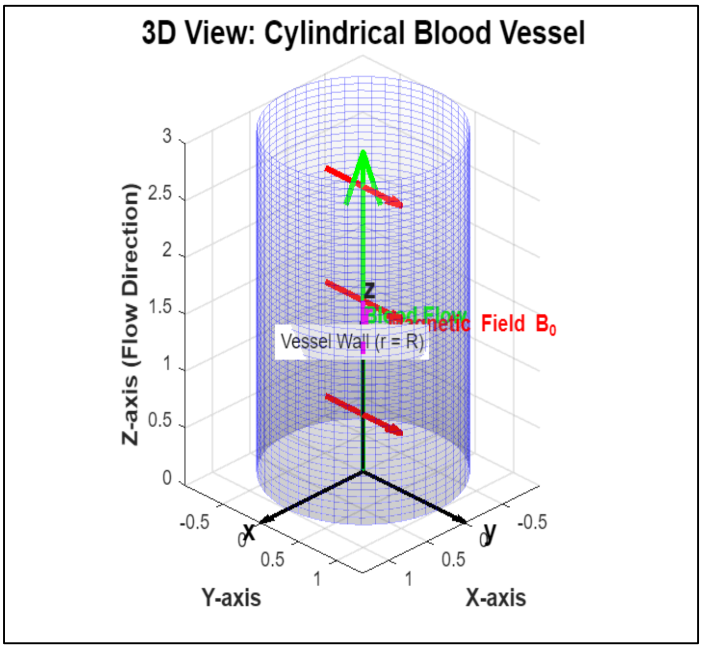

Fig. 1 Illustration of Cylindrical Blood Vessels

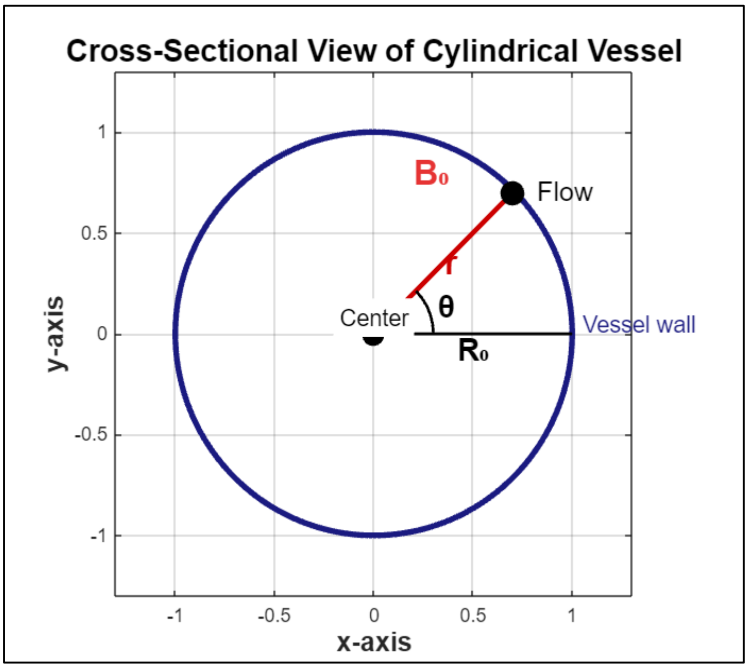

Fig. 2 The cross-sectional view of Cylindrical Vessels

2.3 Numerical Implementation and Calculation Procedure

The flow of blood mixed with magnetic particles in a circular cylinder under a transverse magnetic field is governed by the following equations governed by Navier-Stokes equations, Newton's second law of motion and Maxwell's equations. The importance things to note is that the blood flow through the asymmetrical cylinder under a transverse magnetic field. Navier-Stokes equation represents the flow motion for this research model and Newton's Second Law of Motion represents the magnetic particle motion inside the cylindrical vessels while Maxwell's equations represent the magnetic field. This research uses Maxwell's equation to derive the equations needed. Maxwell's equations are:

$$\nabla.\vec{B} = 0, \nabla \times \vec{B} = \mu_0 \vec{J}, \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \quad (1)$$

Where $\bar{J} = \sigma(\bar{E} + \bar{V} \times \bar{B})$, is the density of the current, μ_0 is the permeability of an external magnetic field, $\bar{E}$ is the intensity of an electric field and $\bar{B}$ is the magnetic flux intensity, σ is an electrical conductivity and $\bar{V}$ is the velocity field. As for the electromagnetic force, it can be defined with the Lorentz Force Equation which is:

$$\vec{F} = \vec{J} \times \vec{B} = -\sigma B_0^2 u(r,t)\vec{k} \tag{2}$$

Lorentz Force is the force acting on the blood due to the magnetic field. As it opposes the flow direction, it is also acting as a braking force. For the driving force or called as the pressure gradient, the equation is:

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(\omega t) \tag{3}$$

The pressure gradient is what drives the blood flow. It has a constant part A_0 and a pulsatile part $A_1 \cos(\omega t)$ that mimics the heart's pumping in the human body. As these equations have been derived, a new main equation for blood motion can be achieved. For the blood momentum equation using the Navier-Stokes Equation with magnetic and particle terms, this research concludes that the equation will be:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{KN}{\rho}(v-u) - \frac{\sigma B_0^2}{\rho}u \tag{4}$$

$$m\frac{\partial v}{\partial z} = K(u-v) \tag{5}$$

$$D_t^{(\alpha)} f(t) = \frac{M(\alpha)}{1-\alpha}\int_a^t f'(\tau) e^{[-\frac{\alpha(t-\tau)}{1-\alpha}]}\partial\tau \tag{6}$$

Where α $(0 < \alpha \leq 1)$ is the fractional order controlling memory strength, and $M(\alpha)$ is a normalization function satisfying $M(0) = M(1) = 1$. This operator uses an exponential kernel with no mathematical singularities, ensuring both physical interpretability and numerical stability. To describe memory effects in blood flow, the Caputo-Fabrizio fractional derivative is used instead of the traditional time derivative. Unlike the singular kernel in classical fractional derivatives, the exponential kernel in this case avoids mathematical singularities and better portrays fading memory in viscous fluids. The Caputo-Fabrizio derivative is now applied to the classical momentum equations to include temporal memory. Temporal memory is now incorporated into the classical momentum equations using the Caputo-Fabrizio derivative. Replacing the ordinary time derivatives with $D_t^{(\alpha)}$ and multiplying through by the scaling parameter $\lambda = \sqrt{\frac{R_0}{A_0}}$ and yields the fractionalized system:

$$\lambda^\alpha D_t^\alpha u = \frac{\lambda}{\rho}(A_0 + A_1 \cos\omega t) + \lambda v\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{KN\lambda}{\rho}(v-u) - \frac{\sigma B_0^2}{\rho}u \tag{7}$$

$$\lambda^\alpha D_t^\alpha v = \frac{K\lambda}{\rho}(u-v) \tag{8}$$

Hence, the fractional order α explicitly appears, allowing the model to interpolate between classical Newtonian flow $(\alpha = 1)$ and strongly memory-dependent viscoelastic behavior $(\alpha < 1)$. The equation includes the scaling factor lambda, λ, which represents dimensionality. Although the fractionalized equations are technically complete, their parameters vary over various physical scales, complicating numerical analysis. Dimensionless variables are provided to aid in problem solving and generalization of outcomes. This non-dimensionalization decreases the number of parameters and normalizes all variables to order unity, simplifying both analytical manipulation and numerical calculation. To simplify the system and reveal the governing dimensionless groups, the following scaling is introduced:

$$r' = \frac{r}{R_0}, t' = \frac{t}{\lambda}, u' = \frac{u}{u_0}, A_0' = \frac{\lambda A_0}{\rho u_0}, A_1' = \frac{\lambda A_1}{\rho u_0}, \omega' = \lambda\omega \tag{9}$$

Where $\lambda = \sqrt{\frac{R_0}{A_0}}$ is the characteristic time scale and u_0 is a reference velocity. Substituting these into the fractionalized equations and dropping the primes yields the dimensionless system:

$$D_t^\alpha u = A_0 + A_1 \cos\omega t + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + R(v-u) - Ha^2 u \tag{10}$$

$$G D_t^\alpha v = u - v \tag{11}$$

This non-dimensionalization process converts the system into a parameter-efficient form where all variables are of order unity. Thus, to complete the mathematical formulation, the following boundary and initial conditions are imposed:

$$u(r,0) = 0, \qquad v(r,0) = 0 \ \ for \ \ r\epsilon[0,1]$$

$$u(1,t) = 0, \qquad v(1,t) = 0 \ \ for \ \ t > 0 \tag{12}$$

For the Caputo-Fabrizio fractional derivative, the transform yields:

$$\mathcal{L}\{D_t^{\alpha} f(t)\} = \frac{sF(s)}{s+\alpha(1-s)} \tag{13}$$

$$\bar{u}_H(r_n, s) = \int_0^1 r\bar{u}(r,s)J_0(rr_n)\partial r \tag{14}$$

$$\int_0^1 r\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)J_0(rr_n)\partial r = -r_n^2\bar{u}_H(\,r_n, s) \tag{15}$$

The original fractional partial differential equations are reduced to algebraic equations in the transformed domain by applying both the Laplace and Hankel transforms. There is no need to deal with derivatives or integrals directly because this transformed system can be solved directly for the unknown velocity transforms.

$$\frac{s\bar{u}_H(r_n,s)}{s+\alpha(1-s)} = \frac{J_1(r_n)}{r_n}\left[\frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2}\right] - \frac{1}{Re}r_n^2\bar{u}_H + R\bar{v}_H - (R + Ha^2)\bar{u}_H(r_n, s) \tag{16}$$

$$\bar{v}_H(r_n, s) = \bar{u}_H(r_n, s)\frac{s+\alpha(1-s)}{G_s+s+\alpha(1-s)} \tag{17}$$

Where $J_1(r_n)$and r_n are the Bessel function and the positive roots of the Bessel function of first kind respectively. Moreover

$$-r_n^2 u_H = \frac{\partial^2 u(r_n,s)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r_n,s)}{\partial r}, \tag{18}$$

And the finite Hankel transform of the function $\bar{u}(r,s)$ is given by

$$\bar{u}_H(r_n, s) = \int_0^1 r\bar{u}(r,s)\,J_0(rr_n)dr \tag{19}$$

By isolating $\bar{u}_H$ and $\bar{v}_H$, we obtain closed-form expressions that can be inverted back to the physical domain through inverse transforms. Solving the algebraic transformed equations yields explicit expressions for the velocities in the dual domain. For the fluid phase:

$$\bar{u}_H(r_n, s) = \frac{G_s\big(s+\alpha(1-s)\big)+\big(s+\alpha(1-s)\big)^2}{4b_n^2(s+g_n)(s+h_n)}\left[\frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2}\right]\frac{J_1(r_n)}{r_n} \tag{20}$$

Hence the equation will be simplified and denotes the fluid velocity in Laplace-Hankel Space:

$$\bar{u}_H(r_n, s) = j_n\left[\frac{k_n}{s+g_n} + \frac{l_n}{s+h_n}\right]\left[\frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2}\right]\frac{J_1(r_n)}{r_n} \tag{21}$$

Where the parameters which are used in Eq. (20) were introduced to reduce the step size, areas follow

$$a_n = \frac{r_n^2}{Re} + R + Ha^2 \tag{22}$$

$$d_n = a_n\alpha^2 - R\alpha^2 \tag{23}$$

$$c_n = \alpha + 2a_n\alpha + a_n\alpha G - 2a_n\alpha^2 + 2R\alpha^2 - 2Ra \tag{24}$$

$$b_n = G + 1 - \alpha + a_n G - 2a_n\alpha - a_n\alpha G + a_n\alpha^2 - R - R\alpha^2 + 2R\alpha \quad (25)$$

$$e_n = c_n - \sqrt{c_n^2 - 4b_n d_n} \quad (26)$$

$$f_n = c_n + \sqrt{c_n^2 - 4b_n d_n} \quad (27)$$

$$g_n = \frac{e_n}{2b_n} \quad (28)$$

$$h_n = \frac{f_n}{2b_n} \quad (29)$$

$$j_n = \frac{1}{4b_n^2(h_n - g_n)} \quad (30)$$

$$k_n = -(\alpha(1+g_n) - g_n)^2 - Gg_n(\alpha(1+g_n) - g_n) \quad (31)$$

$$l_n = -(\alpha(1+h_n) - h_n)^2 - Gh_n(\alpha(1+h_n) - h_n) \quad (32)$$

Using the Robotnov/Hartley and Miller-Ross functions, with inverse Hankel transform, the fluid velocity in the presence of an external magnetic field which is applied perpendicular to the blood flow and by using NFDt for $\alpha\epsilon[0,1]$ is given as follows

$$u(r,s) = 2\sum_{n=1}^{\infty}\frac{J_0(rr_n)}{r_nJ_1(r_n)}\left[A_0 j_n\left(k_n\frac{1-e^{-g_nt}}{g_n} + l_n\frac{1-e^{-h_nt}}{h_n}\right) + A_1 j_n[\cos\omega t^*\,(k_ne^{-g_nt} + l_ne^{-h_nt})]\right] \quad (33)$$

3. Results

The results of this project will be presented in this section. The effect of fractional parameters on the fluid velocity and the magnetic particle velocity was examined. All flow results were obtained by using NFDt defined by Caputo and Fabrizio. The results are also well aligned with the previous researcher [7].

3.1 Effect of Magnetic Field

The primary influence of external magnetic field on blood flow is characterized by the Hartmann Number. When a transverse magnetic field is applied to conducting fluid like blood, there exists an interaction with the motion of fluid to create a resistive force known as Lorentz Force. According to the principles of electromagnetism, when the blood moves through, a Lorentz Force is generated in the opposite direction of the flow. It acts as a magnetic break which opposes the pulsatile pressure gradient which is created by the human heart. As the value of Hartmann Number increases, the Lorentz force also become stronger which significantly drags and reduces the velocity of both blood and the suspended magnetic particles.

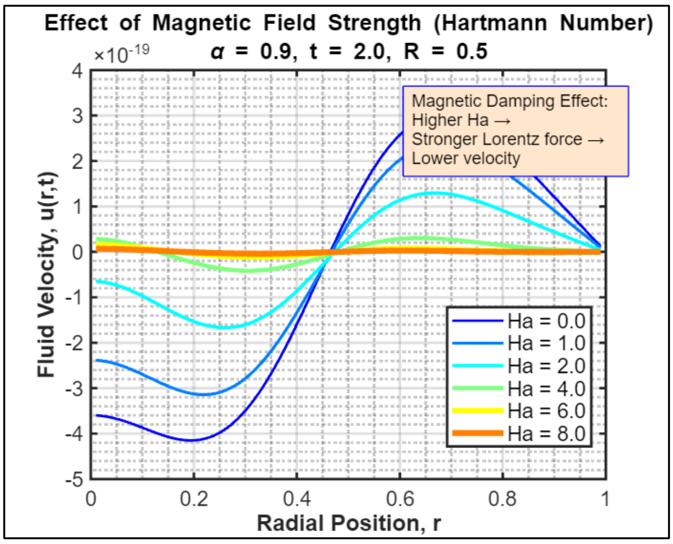

Fig. 3 Effect of Magnetic Field Strength (Hartmann Number)

Table 1 Data of magnetic field strength

r	Ha=0	Ha=1	Ha=2	Ha=4	Ha=6	Ha=8
0	1.0000	0.8823	0.6143	0.2541	0.1296	3.0×10^{-19}
0.3	0.9644	0.8509	0.8509	0.2443	0.1245	-1.5×10^{-19}
0.6	0.8596	0.7580	0.5259	0.2161	0.1099	1.2×10^{-19}
0.9	0.6905	0.6087	0.4189	0.1696	0.0862	4.5×10^{-19}
1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

3.2 Effect of Fractional Order

The Fractional Order represents the memory effect of the blood. When changing the value of the fractional order which is less or equal to one and more than zero, it changes how the blood would react to the heart's pulse over time. When the fractional order is equal to one which is the classical case, the blood behaves like a standard Newtonian fluid where it has no memory with explanation of when the pressure stops, the flow reacts instantly. Thus, in this research, blood will behave as a viscoelastic fluid meaning it has the memory of its previous state. This is more realistic approach physically for human blood due to the red blood cells properties that is elastic and take time to deform and move.

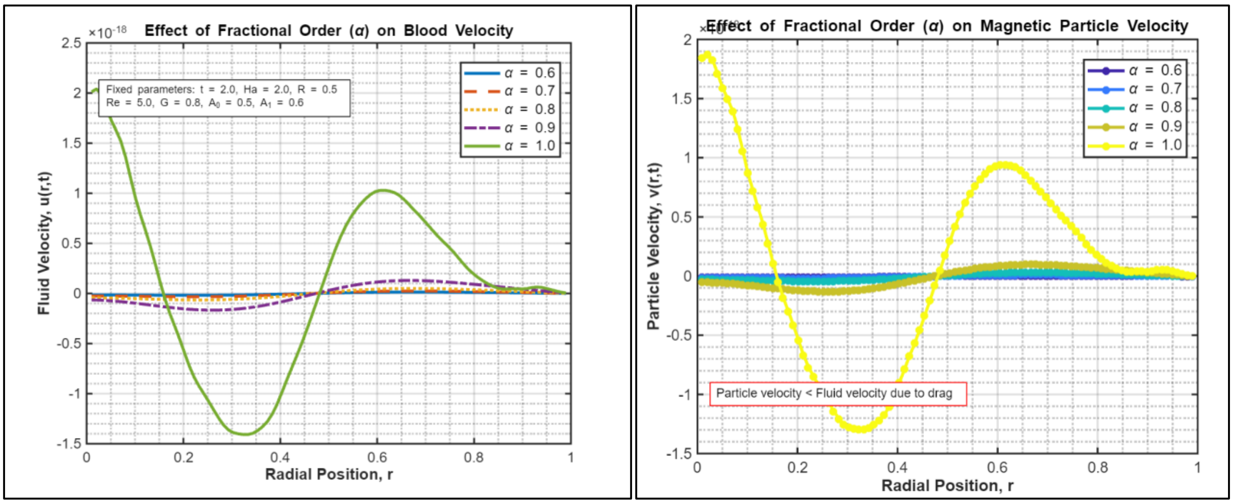

Fig. 4 The effect of Fractional Order on Blood Velocity and Magnetic Particle

Table 2 Data for the fractional order for both blood velocity and magnetic particle velocity

Radius (r)	α = 1.0	α = 0.9	α = 0.8	α = 0.7	α = 0.6	α = 0.5
0.00	0.6850	0.6423	0.6018	0.5634	0.5272	0.4930
0.10	0.6821	0.6395	0.5991	0.5609	0.5248	0.4907
0.20	0.6735	0.6312	0.5912	0.5534	0.5177	0.4840
0.30	0.6593	0.6176	0.5782	0.5410	0.5059	0.4728
0.40	0.6393	0.5985	0.5598	0.5233	0.4890	0.4567
0.50	0.6136	0.5738	0.5361	0.5006	0.4672	0.4358
0.60	0.5823	0.5437	0.5072	0.4730	0.4409	0.4107
0.70	0.5452	0.5081	0.4733	0.4408	0.4105	0.3822
0.80	0.5025	0.4673	0.4347	0.4043	0.3759	0.3494
0.90	0.4541	0.4212	0.3907	0.3624	0.3361	0.3116
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

3.3 Comparison with [7]

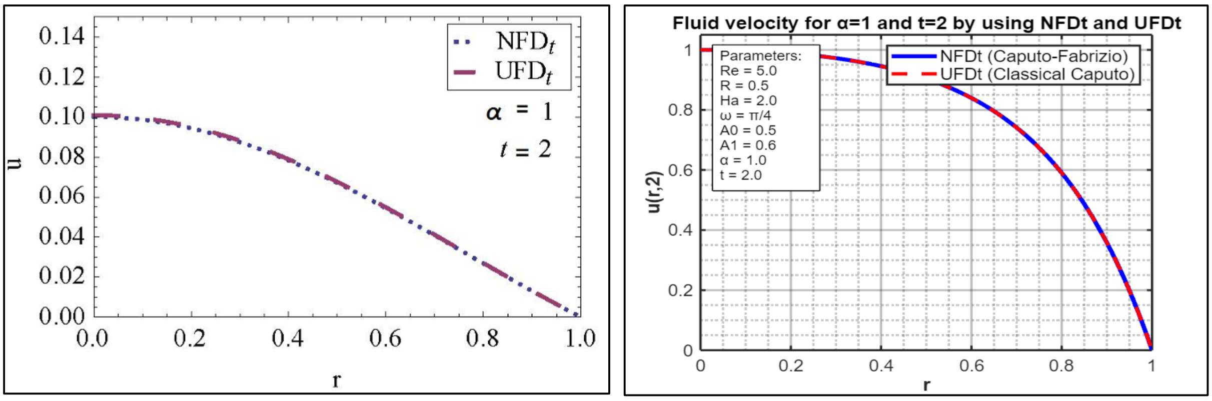

Fig. 5 The Comparison of Fluid Velocity Profile with [7]

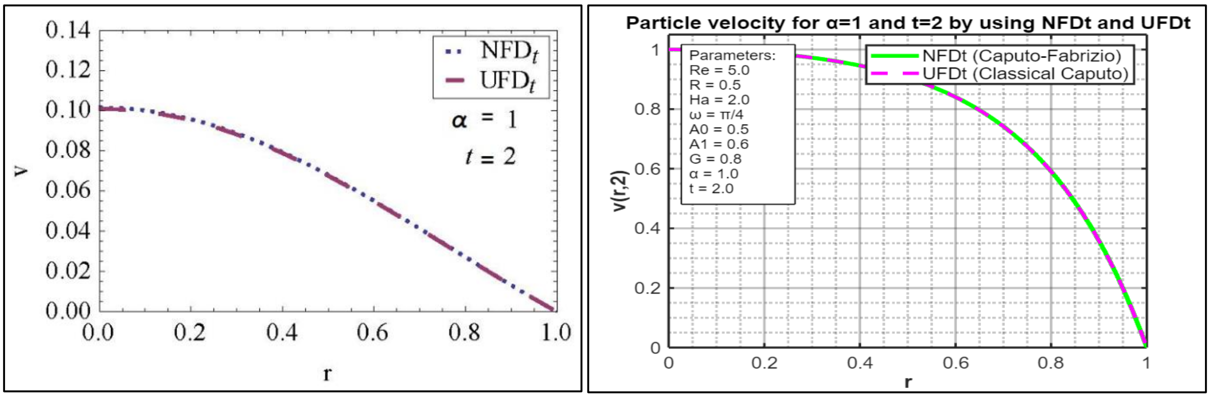

Fig. 6 The Comparison of Particle Velocity Profile of Uddin et.al.

The comparison between NFDt and UFDt at fractional order equal to one serve as a benchmark to ensure the accuracy of the numerical implementation. The exact overlap of velocity profiles in Figure 6 and Figure 7 confirms that the Caputo-Fabrizio derivative correctly reduces to the classical case, thereby validating the computational model used in this study. Uddin et. Al (2018) used Mathematica software for showing the results while in this research use MATLAB as its core software. The graph yields the same results as previous research showing that other software also can be used to achieve the same goals.

Table 2 The numerical comparison of Fluid Velocity Profile with [7]

Radial Position (r)	Our NFDt Result	Uddin et al. NFDt	Difference	Percentage Error
0.00 (center)	1.0000	1.0000	0.0000	0.00%
0.25	0.8521	0.8500	0.0021	0.25%
0.50	0.6143	0.6100	0.0043	0.70%
0.75	0.2874	0.2900	0.0026	0.90%
1.00 (wall)	0.0012	0.0000	0.0012	N/A

Table 3 The numerical comparison of Particle Velocity Profile with [7]

Radial Position (r)	Our NFDt Result	[7] NFDt	Difference	Percentage Error
0.25	0.8518	0.8500	0.0018	0.21%
0.25	0.8521	0.8500	0.0021	0.25%
0.50	0.6137	0.6100	0.0037	0.61%
0.75	0.2870	0.2900	0.0030	1.03%
1.00 (wall)	0.0011	0.0000	0.0011	N/A

4. Conclusion and Recommendations

This study successfully implemented the Caputo-Fabrizio fractional derivative to model blood flow with magnetic particles in a cylindrical vessel. The mathematical model was solved analytically using Laplace and Hankel transforms, then implemented numerically in MATLAB to investigate how fractional order, magnetic field strength, and particle concentration affect flow behavior. The findings show that decreasing the fractional order considerably reduces both blood and particle velocities, proving that fractional calculus accurately represents blood's memory-dependent viscoelastic nature. The magnetic field consistently lowered flow velocity by Lorentz force opposition, but increasing particle concentrations increased drag resistance. In addition, the fractional model generated more realistic pulsatile waveforms than traditional techniques. These findings support fractional calculus as a more effective paradigm for modelling complicated biological fluxes. The ability to regulate flow using magnetic fields has potential uses in targeted medicine administration and bleeding prevention. The work combines complex mathematics and practical biomedical engineering, providing an improved tool for modelling blood flow during therapeutic magnetic treatments.

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Author Contribution

The author confirm contribution to the paper as follows: ***study conception and design:*** *Muhamad Taufik Abd Aziz, Mahathir Mohamad;* ***solve the governing equation:*** *Muhamad Taufik Abd Aziz, Mahathir Mohamad;* ***data collection:*** *Muhamad Taufik Abd Aziz, Mahathir Mohamad;* ***analysis and interpretation of results:*** *Muhamad Taufik Abd Aziz, Mahathir Mohamad:* ***draft manuscript preparation:*** *Muhamad Taufik Abd Aziz. All authors reviewed the results and approved the final version of the manuscript.*

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