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Dormand-Prince Method for Solving SIR Model for Hand, Foot and Mouth Disease in Sarawak

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Abstract

Mathematical modelling is essential in understanding the transmission of Hand, Foot and Mouth Disease (HFMD), which remains a considerable public health issue, especially among children. The aim of this study is to understand the transmission dynamics of HFMD in Sarawak using the Susceptible-Infected-Recovered (SIR) model as its numerical approach. This study focuses on solving the system of ordinary differential equations controlling the SIR model using the Dormand-Prince method (RK45). MATLAB was utilized to perform numerical simulations of the SIR model using real epidemiological data. Three different step size h = 0.01,0.05 and 0.1 used to see the effect of step size to the numerical obtain when RK45 applied. The numerical result show that the RK45 produces stable and consistent solutions. The RK45 also successfully demonstrate the transmission dynamics of HFMD. In conclusion, the results prove that the RK45 is an effective numerical approach for solving the SIR model of HFMD in Sarawak.

1. Introduction

Hand, Foot and Mouth Disease (HFMD) is one of the common infectious diseases that mostly impact young children but cases in adults also have been reported [1],[2]. HFMD is caused by a group of enteroviruses, including Coxsackievirus A16 (CA16) and Enterovirus 71 (EV71), which are known for their potential to cause large-scale outbreaks and severe complications in children [3]. The World Health Organization (WHO) states that EV71 infections are linked to neurological complications and fatal outcomes in severe cases and poses a significant public health concern [4]. The common symptoms of the disease are fever, painful oral lesions and vesicular rashes on the hands and feet which are spread by direct contact with bodily secretions of infected people [5].

HFMD cases frequently reported in Western Pacific Region including Malaysia [4]. Sarawak consistently has greatest number of HFMD cases in Malaysia due to the frequent HFMD outbreaks [6]. Major outbreaks as those recorded in 1997 and subsequent years emphasize the necessity of effective monitoring and intervention strategies to control the development of HFMD. These recurrent outbreaks highlight how important it is to understand the dynamics of HFMD transmission utilizing computational and mathematical approaches.

The Susceptible-Infected-Recovered (SIR) model is one of the most used frameworks in mathematical modelling for analysing the spread of infectious illnesses. The SIR model is formulated as a system of nonlinear ordinary differential equations (ODEs), which are typically challenging to solve, and numerical techniques are required to solve the system. Due to stability, classical methods such as the fourth-order Runge-Kutta (RK4) method widely employed [7],[8]. However, adaptive step size techniques, like the Runge-Kutta-Fehlberg (RKF45) and the Dormand-Prince method (RK45) offer better numerical accuracy and efficiency by enabling step sizes to be automatically adjusted depending on error estimation [9],[10],[11]. For adaptive step size selection, the RK45 was designed as an embedded Runge-Kutta pair that provides both fourth-and fifth order estimates [10],[11].

The performance of the RK45 in solving the SIR model of HFMD with actual epidemiological parameters has not been extensively studied, although the fact that many studies have applied Runge-Kutta method to epidemic models [7],[8]. This gap necessitates additional assessment of RK45's numerical accuracy and efficiency for HFMD modelling. Therefore, this study applied RK45 for solving the SIR model for HFMD in Sarawak.

There are two objectives in this study, the first objective is to understand the transmission of HFMD using the SIR model. The second objective of this study is to solve the SIR model of HFMD using the RK45. MATLAB is utilizing to implement numerical simulation to analyses the dynamics of HFMD and evaluate the effectiveness of RK45 in solving the SIR model. This study used three compartment SIR frameworks considering natural birth and death rates with epidemiological data adopted from previous studies.

2. Methodology

This section describes the SIR model of HFMD used to represent the transmission of HFMD in Sarawak and the RK45 method used to solve the model. The system of ODEs is solved using the RK45 implemented by ode45 solver in MATLAB. Three step size h =0.01, 0.05 and 0.1 are used in numerical simulation. Mean Error Analysis (MAE) is performed to evaluate the accuracy of the numerical solutions.

2.1 SIR Model of HMFD

Fig. 1 represents the SIR model of HFMD in Sarawak that was used for this study to understand the dynamics transmission of the disease. The population is divided into three groups which are susceptible, infected and recovered.

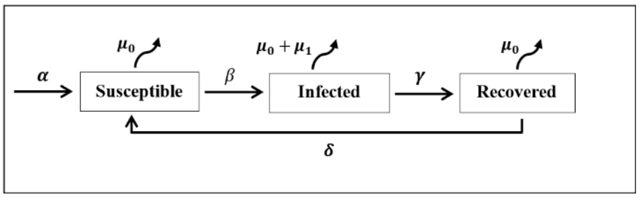


Fig. 1 SIR Model of HMFD [12]

Table 1

Table 1 The parameter and definition in the SIR model

Parameter	Definition
S	The number of susceptible individuals
I	The number of infected individuals
R	The number of recovered individuals
α	The natural birth rate
β	The transmission coefficient
γ	The recovery rate of an infectious individual
δ	The immunity loss rate of a recovered individual
μ_0	The natural death rate
μ_1	The death rate due to disease

Table 1 assembled all the parameters and its definitions that used to describe the SIR model of HFMD. Important epidemiological processes are expressed by the parameters. The numerical values of the SIR model are adopted from previous study [12]. The value of $\alpha = 0.0002923, \beta = 0.00015, \gamma = 0.8235, \delta = 0.07, \mu_0 = 0.0001077$, and $\mu_1 = 0.00001731$. The mathematical formulation of the SIR model used all these parameters which play an important role in understanding the transmission.

The system of ODEs that can be obtained from the SIR model of HFMD in this study were taken from previous work [12] is given by:

$$\frac{dS}{dt}=\alpha S(t)-\beta I(t)S(t)-\mu_0 S(t)+\delta R(t) \tag{1}$$

$$\frac{dI}{dt}=\beta I(t)S(t)-\gamma I(t)-(\mu_0+\mu_1)I(t) \tag{2}$$

$$\frac{dR}{dt}=\gamma I(t)-\delta R(t)-\mu_0 R(t) \tag{3}$$

To solve the SIR model numerically, initial conditions are required. The initial conditions represent the population distribution at the starting time of the simulation, Specifically, the initial S population is 10000, the initial I population is 4 and the initial R population is 0 that obtained from previous study [12]. These initial conditions serve as the starting point for the numerical solution of the SIR model.

2.2 Dormand-Prince Method

To solve the system of ODEs in the SIR model, the RK45 was used in this study. Two approaches were applied. The first approach is the fixed step size of *h* = 0.01, 0.05 and 0.1 to examine the impact of step size on numerical accuracy. The second approach is the adaptive-step-size RK45 which automatically modifies the step size according to error estimation and is implemented in MATLAB using the ode45 solver. Based on the Butcher tableau of the RK45, the function evaluation of each stage is calculated to get the numerical approximation of the next stage [9]. The RK45 was applied to each of the compartmental population in the SIR model.

The numerical approximation of susceptible populations at the next step is given:

$$S_{n+1}=S_n+\frac{35}{384}k_1^S+\frac{500}{1113}k_3^S+\frac{125}{192}k_4^S-\frac{2187}{6784}k_5^S+\frac{11}{84}k_6^S \tag{4}$$

where the coefficient of k_1^S to k_7^S defined as:

$$k_1^S=hf\left(t_n,S_n,I_n,R_n\right)=h\left(\alpha S_n-\beta I_n S_n-\mu_0 S_n+\delta R_n\right) \tag{5}$$

$$\begin{aligned}k_2^S&=hf\left(t_n+\frac{1}{5}h,S_n+\frac{1}{5}k_1^S,I_n+\frac{1}{5}k_1^I,R_n+\frac{1}{5}k_1^R\right)\\&=h\left(\alpha\left(S_n+\frac{1}{5}k_1^S\right)-\beta\left(I_n+\frac{1}{5}k_1^I\right)\left(S_n+\frac{1}{5}k_1^S\right)-\mu_0\left(S_n+\frac{1}{5}k_1^S\right)+\delta\left(R_n+\frac{1}{5}k_1^R\right)\right)\end{aligned} \tag{6}$$

$$\begin{aligned}k_3^S&=hf\left(t_n+\frac{3}{10}h,S_n+\frac{3}{40}k_1^S\frac{9}{40}k_2^S,I_n+\frac{3}{40}k_1^I\frac{9}{40}k_2^I,R_n+\frac{3}{40}k_1^R\frac{9}{40}k_2^R\right)\\&=h\begin{pmatrix}\alpha\left(S_n+\frac{3}{40}k_1^S\frac{9}{40}k_2^S\right)-\beta\left(I_n+\frac{3}{40}k_1^I\frac{9}{40}k_2^I\right)\left(S_n+\frac{3}{40}k_1^S\frac{9}{40}k_2^S\right)\\-\mu_0\left(S_n+\frac{3}{40}k_1^S\frac{9}{40}k_2^S\right)+\delta\left(R_n+\frac{3}{40}k_1^R\frac{9}{40}k_2^R\right)\end{pmatrix}\end{aligned} \tag{7}$$

$$k_4^S = hf\begin{pmatrix} t_n + \frac{4}{5}h, S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S, I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I, \\ R_n + \frac{44}{45}k_1^R - \frac{56}{15}k_2^R + \frac{32}{9}k_3^R \end{pmatrix}$$

$$= h\begin{pmatrix} \alpha\left(S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S\right) - \beta\left(I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \\ \left(S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S\right) - \mu_0\left(S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S\right) \\ +\delta\left(R_n + \frac{44}{45}k_1^R - \frac{56}{15}k_2^R + \frac{32}{9}k_3^R\right) \end{pmatrix} \quad (8)$$

$$k_5^S = hf\begin{pmatrix} t_n + \frac{8}{9}h, S_n + \frac{19372}{6561}k_1^S - \frac{25360}{2187}k_2^S + \frac{64448}{6561}k_3^S - \frac{212}{729}k_4^S, \\ I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I, \\ R_n + \frac{19372}{6561}k_1^R - \frac{25360}{2187}k_2^R + \frac{64448}{6561}k_3^R - \frac{212}{729}k_4^R, \end{pmatrix}$$

$$= h\begin{pmatrix} \alpha\left(S_n + \frac{19372}{6561}k_1^S - \frac{25360}{2187}k_2^S + \frac{64448}{6561}k_3^S - \frac{212}{729}k_4^S\right) \\ -\beta\left(I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I\right) \\ \left(S_n + \frac{19372}{6561}k_1^S - \frac{25360}{2187}k_2^S + \frac{64448}{6561}k_3^S - \frac{212}{729}k_4^S\right) \\ -\mu_0\left(S_n + \frac{19372}{6561}k_1^S - \frac{25360}{2187}k_2^S + \frac{64448}{6561}k_3^S - \frac{212}{729}k_4^S\right) \\ +\delta\left(R_n + \frac{19372}{6561}k_1^R - \frac{25360}{2187}k_2^R + \frac{64448}{6561}k_3^R - \frac{212}{729}k_4^R\right) \end{pmatrix} \quad (9)$$

$$k_6^S = hf\begin{pmatrix} t_n + h, S_n + \frac{9017}{3168}k_1^S - \frac{355}{33}k_2^S + \frac{46732}{5247}k_3^S + \frac{49}{176}k_4^S - \frac{5103}{18656}k_5^S, \\ I_n + \frac{9017}{3168}k_1^I - \frac{355}{33}k_2^I + \frac{46732}{5247}k_3^I + \frac{49}{176}k_4^I - \frac{5103}{18656}k_5^I, \\ R_n + \frac{9017}{3168}k_1^R - \frac{355}{33}k_2^R + \frac{46732}{5247}k_3^R + \frac{49}{176}k_4^R - \frac{5103}{18656}k_5^R \end{pmatrix}$$

$$= h\begin{pmatrix} \alpha\left(S_n + \frac{9017}{3168}k_1^S - \frac{355}{33}k_2^S + \frac{46732}{5247}k_3^S + \frac{49}{176}k_4^S - \frac{5103}{18656}k_5^S\right) \\ -\beta\left(I_n + \frac{9017}{3168}k_1^I - \frac{355}{33}k_2^I + \frac{46732}{5247}k_3^I + \frac{49}{176}k_4^I - \frac{5103}{18656}k_5^I\right) \\ \left(S_n + \frac{9017}{3168}k_1^S - \frac{355}{33}k_2^S + \frac{46732}{5247}k_3^S + \frac{49}{176}k_4^S - \frac{5103}{18656}k_5^S\right) \\ -\mu_0\left(S_n + \frac{9017}{3168}k_1^S - \frac{355}{33}k_2^S + \frac{46732}{5247}k_3^S + \frac{49}{176}k_4^S - \frac{5103}{18656}k_5^S\right) \\ +\delta\left(R_n + \frac{9017}{3168}k_1^R - \frac{355}{33}k_2^R + \frac{46732}{5247}k_3^R + \frac{49}{176}k_4^R - \frac{5103}{18656}k_5^R\right) \end{pmatrix} \quad (10)$$

$$
\begin{aligned}
k_7^S &= hf\begin{pmatrix} t_n + h, S_n + \frac{35}{384}k_1^S + \frac{500}{1113}k_3^S + \frac{125}{192}k_4^S - \frac{2187}{6784}k_5^S + \frac{11}{84}k_6^S, \\ I_n + \frac{35}{384}k_1^I + \frac{500}{1113}k_3^I + \frac{125}{192}k_4^I - \frac{2187}{6784}k_5^I + \frac{11}{84}k_6^I, \\ R_n + \frac{35}{384}k_1^R + \frac{500}{1113}k_3^R + \frac{125}{192}k_4^R - \frac{2187}{6784}k_5^R + \frac{11}{84}k_6^R \end{pmatrix} \\
&= h\begin{pmatrix} \alpha\left(S_n + \frac{35}{384}k_1^S + \frac{500}{1113}k_3^S + \frac{125}{192}k_4^S - \frac{2187}{6784}k_5^S + \frac{11}{84}k_6^S\right) \\ -\beta\left(I_n + \frac{35}{384}k_1^I + \frac{500}{1113}k_3^I + \frac{125}{192}k_4^I - \frac{2187}{6784}k_5^I + \frac{11}{84}k_6^I\right) \\ \left(S_n + \frac{35}{384}k_1^S + \frac{500}{1113}k_3^S + \frac{125}{192}k_4^S - \frac{2187}{6784}k_5^S + \frac{11}{84}k_6^S\right) \\ -\mu_0\left(S_n + \frac{35}{384}k_1^S + \frac{500}{1113}k_3^S + \frac{125}{192}k_4^S - \frac{2187}{6784}k_5^S + \frac{11}{84}k_6^S\right) \\ +\delta\left(R_n + \frac{35}{384}k_1^R + \frac{500}{1113}k_3^R + \frac{125}{192}k_4^R - \frac{2187}{6784}k_5^R + \frac{11}{84}k_6^R\right) \end{pmatrix}
\end{aligned} \tag{11}
$$

The numerical approximation of infected population at the next step is given:

$$
I_{n+1} = I_n + \frac{35}{384}k_1^I + \frac{500}{1113}k_3^I + \frac{125}{192}k_4^I - \frac{2187}{6784}k_5^I + \frac{11}{84}k_6^I \tag{12}
$$

where the coefficient of k_1^I to k_7^I defined as:

$$
k_1^I = hf\left(t_n, S_n, I_n\right) = h\left(\beta I_n S_n - \gamma I_n - (\mu_0 + \mu_1) I_n\right) \tag{13}
$$

$$
\begin{aligned}
k_2^I &= hf\left(t_n + \frac{1}{5}h, S_n + \frac{1}{5}k_1^S, I_n + \frac{1}{5}k_1^I\right) \\
&= h\left(\beta\left(I_n + \frac{1}{5}k_1^I\right)\left(S_n + \frac{1}{5}k_1^S\right) - \gamma\left(I_n + \frac{1}{5}k_1^I\right) - (\mu_0 + \mu_1)\left(I_n + \frac{1}{5}k_1^I\right)\right)
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
k_3^I &= hf\left(t_n + \frac{3}{10}h, S_n + \frac{3}{40}k_1^S \frac{9}{40}k_2^S, I_n + \frac{3}{40}k_1^I \frac{9}{40}k_2^I\right) \\
&= h\begin{pmatrix} \beta\left(I_n + \frac{3}{40}k_1^I \frac{9}{40}k_2^I\right)\left(S_n + \frac{1}{5}k_1^S S_n + \frac{3}{40}k_1^S \frac{9}{40}k_2^S\right) \\ -\gamma\left(I_n + \frac{3}{40}k_1^I \frac{9}{40}k_2^I\right) - (\mu_0 + \mu_1)\left(I_n + \frac{3}{40}k_1^I \frac{9}{40}k_2^I\right) \end{pmatrix}
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
k_4^I &= hf\left(t_i + \frac{4}{5}h, S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S, I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \\
&= h\begin{pmatrix} \beta\left(I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right)\left(S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S\right) \\ -\gamma\left(I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \\ -(\mu_0 + \mu_1)\left(I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \end{pmatrix}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
k_5^I &= hf\begin{pmatrix} t_n+\frac{8}{9}h, S_n+\frac{19372}{6561}k_1^S-\frac{25360}{2187}k_2^S+\frac{64448}{6561}k_3^S-\frac{212}{729}k_4^S, \\ I_n+\frac{19372}{6561}k_1^I-\frac{25360}{2187}k_2^I+\frac{64448}{6561}k_3^I-\frac{212}{729}k_4^I \end{pmatrix} \\
&= h\begin{pmatrix} \beta\left(I_n+\frac{19372}{6561}k_1^I-\frac{25360}{2187}k_2^I+\frac{64448}{6561}k_3^I-\frac{212}{729}k_4^I\right) \\ \left(S_n+\frac{19372}{6561}k_1^S-\frac{25360}{2187}k_2^S+\frac{64448}{6561}k_3^S-\frac{212}{729}k_4^S\right) \\ -\gamma\left(I_n+\frac{19372}{6561}k_1^I-\frac{25360}{2187}k_2^I+\frac{64448}{6561}k_3^I-\frac{212}{729}k_4^I\right) \\ -(\mu_0+\mu_1)\left(I_n+\frac{19372}{6561}k_1^I-\frac{25360}{2187}k_2^I+\frac{64448}{6561}k_3^I-\frac{212}{729}k_4^I\right) \end{pmatrix}
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
k_6^I &= hf\begin{pmatrix} t_n+h, S_n+\frac{9017}{3168}k_1^S-\frac{355}{33}k_2^S+\frac{46732}{5247}k_3^S+\frac{49}{176}k_4^S-\frac{5103}{18656}k_5^S, \\ I_n+\frac{9017}{3168}k_1^I-\frac{355}{33}k_2^I+\frac{46732}{5247}k_3^I+\frac{49}{176}k_4^I-\frac{5103}{18656}k_5^I \end{pmatrix} \\
&= h\begin{pmatrix} \beta\left(I_n+\frac{9017}{3168}k_1^I-\frac{355}{33}k_2^I+\frac{46732}{5247}k_3^I+\frac{49}{176}k_4^I-\frac{5103}{18656}k_5^I\right) \\ \left(S_n+\frac{9017}{3168}k_1^S-\frac{355}{33}k_2^S+\frac{46732}{5247}k_3^S+\frac{49}{176}k_4^S-\frac{5103}{18656}k_5^S\right) \\ -\gamma\left(I_n+\frac{9017}{3168}k_1^I-\frac{355}{33}k_2^I+\frac{46732}{5247}k_3^I+\frac{49}{176}k_4^I-\frac{5103}{18656}k_5^I\right) \\ -(\mu_0+\mu_1)\left(I_n+\frac{9017}{3168}k_1^I-\frac{355}{33}k_2^I+\frac{46732}{5247}k_3^I+\frac{49}{176}k_4^I-\frac{5103}{18656}k_5^I\right) \end{pmatrix}
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
k_7^I &= hf\begin{pmatrix} t_n+h, S_n+\frac{35}{384}k_1^S+\frac{500}{1113}k_3^S+\frac{125}{192}k_4^S-\frac{2187}{6784}k_5^S+\frac{11}{84}k_6^S, \\ I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I \end{pmatrix} \\
&= h\begin{pmatrix} \beta\left(I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I\right) \\ \left(S_n+\frac{35}{384}k_1^S+\frac{500}{1113}k_3^S+\frac{125}{192}k_4^S-\frac{2187}{6784}k_5^S+\frac{11}{84}k_6^S\right) \\ -\gamma\left(I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I\right) \\ -(\mu_0+\mu_1)\left(I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I\right) \end{pmatrix}
\end{aligned}
\tag{19}
$$

The numerical approximation of recovered population at the next step is given:

$$
R_{n+1} = R_n+\frac{35}{384}k_1^R+\frac{500}{1113}k_3^R+\frac{125}{192}k_4^R-\frac{2187}{6784}k_5^R+\frac{11}{84}k_6^R \tag{20}
$$

where the coefficient of k_1^R to k_7^R defined as:

$$
k_1^R = hf\left(t_n, I_n, R_n\right) = h\left(\gamma I_n-\delta R_n-\mu_0 R_n\right) \tag{21}
$$

$$k_2^R = hf\left(t_n + \frac{1}{5}h, I_n + \frac{1}{5}k_1^I, R_n + \frac{1}{5}k_1^R\right)$$
$$= h\left(\gamma\left(I_n + \frac{1}{5}k_1^I\right) - \delta\left(R_n + \frac{1}{5}k_1^R\right) - \mu_0\left(R_n + \frac{1}{5}k_1^R\right)\right) \tag{22}$$

$$k_3^R = hf\left(t_n + \frac{3}{10}h, I_n + \frac{3}{40}k_1^I \frac{9}{40}k_2^I, R_n + \frac{3}{40}k_1^R \frac{9}{40}k_2^R\right)$$
$$= h\begin{pmatrix} \gamma\left(I_n + \frac{3}{40}k_1^I \frac{9}{40}k_2^I\right) - \delta\left(R_n + \frac{3}{40}k_1^R \frac{9}{40}k_2^R\right) \\ -\mu_0\left(R_n + \frac{3}{40}k_1^R \frac{9}{40}k_2^R\right) \end{pmatrix} \tag{23}$$

$$k_4^R = hf\begin{pmatrix} t_n + \frac{4}{5}h, I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I, \\ R_n + \frac{44}{45}k_1^R - \frac{56}{15}k_2^R + \frac{32}{9}k_3^R \end{pmatrix}$$
$$= h\begin{pmatrix} \gamma\left(I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) - \delta\left(R_n + \frac{44}{45}k_1^R - \frac{56}{15}k_2^R + \frac{32}{9}k_3^R\right) \\ -\mu_0\left(R_n + \frac{44}{45}k_1^R - \frac{56}{15}k_2^R + \frac{32}{9}k_3^R\right) \end{pmatrix} \tag{24}$$

$$k_5^R = hf\begin{pmatrix} t_n + \frac{8}{9}h, I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I, \\ R_n + \frac{19372}{6561}k_1^R - \frac{25360}{2187}k_2^R + \frac{64448}{6561}k_3^R - \frac{212}{729}k_4^R \end{pmatrix}$$
$$= h\begin{pmatrix} \gamma\left(I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I\right) \\ -\delta\left(R_n + \frac{19372}{6561}k_1^R - \frac{25360}{2187}k_2^R + \frac{64448}{6561}k_3^R - \frac{212}{729}k_4^R\right) \\ -\mu_0\left(R_n + \frac{19372}{6561}k_1^R - \frac{25360}{2187}k_2^R + \frac{64448}{6561}k_3^R - \frac{212}{729}k_4^R\right) \end{pmatrix} \tag{25}$$

$$k_6^R = hf\begin{pmatrix} t_n + h, I_n + \frac{9017}{3168}k_1^I - \frac{355}{33}k_2^I + \frac{46732}{5247}k_3^I + \frac{49}{176}k_4^I - \frac{5103}{18656}k_5^I, \\ R_n + \frac{9017}{3168}k_1^R - \frac{355}{33}k_2^R + \frac{46732}{5247}k_3^R + \frac{49}{176}k_4^R - \frac{5103}{18656}k_5^R \end{pmatrix}$$
$$= h\begin{pmatrix} \gamma\left(I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I\right) \\ -\delta\left(R_n + \frac{9017}{3168}k_1^R - \frac{355}{33}k_2^R + \frac{46732}{5247}k_3^R + \frac{49}{176}k_4^R - \frac{5103}{18656}k_5^R\right) \\ -\mu_0\left(R_n + \frac{9017}{3168}k_1^R - \frac{355}{33}k_2^R + \frac{46732}{5247}k_3^R + \frac{49}{176}k_4^R - \frac{5103}{18656}k_5^R\right) \end{pmatrix} \tag{26}$$

$$k_7^S = hf\begin{pmatrix} t_n + h, I_n + \frac{35}{384}k_1^I + \frac{500}{1113}k_3^I + \frac{125}{192}k_4^I - \frac{2187}{6784}k_5^I + \frac{11}{84}k_6^I, \\ R_n + \frac{35}{384}k_1^R + \frac{500}{1113}k_3^R + \frac{125}{192}k_4^R - \frac{2187}{6784}k_5^R + \frac{11}{84}k_6^R \end{pmatrix}$$

$$= h\begin{pmatrix} \gamma\left(I_n + \frac{35}{384}k_1^I + \frac{500}{1113}k_3^I + \frac{125}{192}k_4^I - \frac{2187}{6784}k_5^I + \frac{11}{84}k_6^I\right) \\ -\delta\left(R_n + \frac{35}{384}k_1^R + \frac{500}{1113}k_3^R + \frac{125}{192}k_4^R - \frac{2187}{6784}k_5^R + \frac{11}{84}k_6^R\right) \\ -\mu_0\left(R_n + \frac{35}{384}k_1^R + \frac{500}{1113}k_3^R + \frac{125}{192}k_4^R - \frac{2187}{6784}k_5^R + \frac{11}{84}k_6^R\right) \end{pmatrix} \quad (27)$$

2.3 Error Analysis

In this study, error analysis was carried out to evaluate the accuracy of the numerical solutions obtained via the RK45 with different step sizes. The MATLAB ode45 solver was employed to generate a reference solution with high accuracy. The ode45 solver is suitable as a benchmark because it implements an adaptive step size with error control [13].

Error analysis was conducted using the Mean Absolute Error (MAE) due to its simplicity and effectiveness in measuring the average magnitude of numerical errors [14]. The MAE was calculated indecently for each compartment of the SIR model. In this study, the error analysis focuses on MAE to assess numerical accuracy, whereas other performance indicators such as Root Mean Square Error (RSME), convergence order and CPU computation time are not included. The MAE was computed based on the following formula:

$$MAE = \left(\frac{1}{n}\right)\sum_{i=1}^{n}\left|y_i^{RK45} - y_i^{ref}\right| \quad (28)$$

where:

y_i^{RK45} is the RK45 fixed-step solution

y_i^{ref} is the ode45 reference solution

n is the number of time points

3. Results and Discussion

3.1 Simulation of SIR Model of HFMD in Sarawak

The simulation result of the SIR model for HFMD in Sarawak obtained using the RK45 was implemented in MATLAB by using the ode45 function. The simulation is designed to provide a better understanding of the disease transmission dynamics while observing the transmission dynamics of the susceptible, infected and recovered populations over time. The simulation period was within 52 weeks.

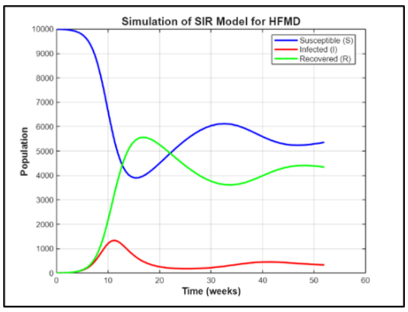


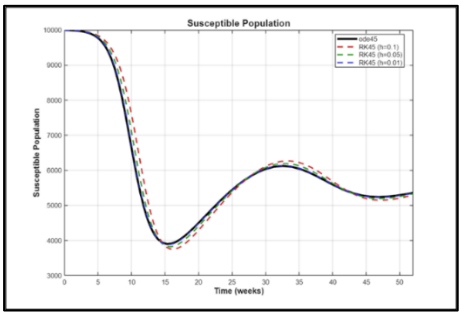
Fig. 2 Simulation of SIR Model for HFMD

Fig. 2 illustrates the simulation of the SIR model for HFMD in Sarawak, and it shows the time evolution of the susceptible, infected and recovered populations for HFMD in Sarawak. The susceptible population is initially high and rapidly declines as more individuals become infected. For instance, during week 10, the susceptible population decreases from 10000 to approximately 6000 individuals yet the infected population increases to more than 1000 individuals, indicating active disease transmission. At the same time, the recovered population rises to over 2000 individuals as infected individuals recover and relocate to the recovered population.

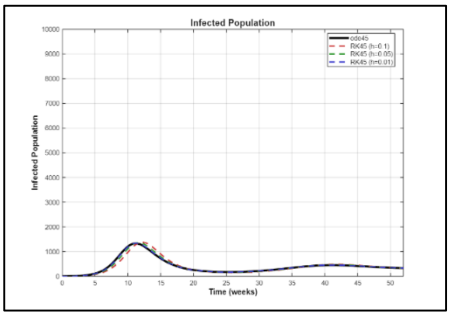
As the recovery process takes precedence over the transmission, the infected population gradually decreases after reaching its peak. Due to immunity loss and demographics factors, the susceptible population demonstrates a slight decrease afterward, while the recovered population shifts cause by reinfection. In addition, considering the model incorporates natural birth and death rates, the total population slightly changes from the initial total population at the beginning of the simulation period. The simulation graph illustrates the dynamic interaction between three groups of the population and provides an understandable visual representation of HFMD in Sarawak transmission. The susceptible, infected and recovered population observed trends are aligned with the theoretical characteristics of the SIR model. This presents that the simulation setup and numerical implementation are appropriate and can provide a reliable foundation for numerical and error analysis.

3.2 Numerical Results

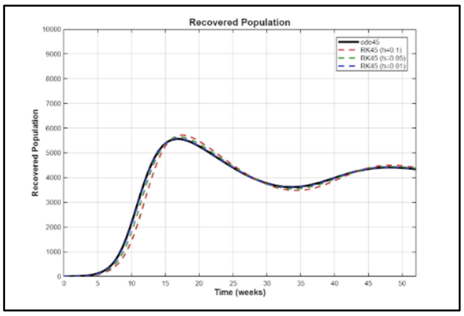
Fig. 3 illustrates the results of RK45 in solving SIR model of HFMD in Sarawak using step size h = 0.01, 0.05 and 0.1. The calculation was carried out in MATLAB.



(a)



(b)



(c)

Fig. 3 Graph of numerical results of the SIR model of HFMD for each compartment population using RK45 with step size h=0.01,0.05 and 0.1 which (a) is susceptible population, (b) is infected population, (c) is recovered population

Table 2 Final population at week 52

Step Size	Final Susceptible Population	Final Infected Population	Final Recovered Population
ode45	5358.55	335.38	4345.64
0.1	5299.31	326.99	4414.67
0.05	5333.45	331.05	4375.71
0.01	5353.89	334.61	4351.19

Table 2 shows the final population values at week 52 obtained the ode45 solver and three different step size. The findings demonstrate the susceptible, infected and recovered populations obtained by h = 0.01,0.05 and 0.1 are close to those generated by ode45 with smaller step size yielding result more in line with reference solution.

3.3 Error Analysis

Fig. 4 demonstrates results of error analysis of the SIR model utilizing the RK45 with different step size h = 0.01, 0.05, and 0.1.

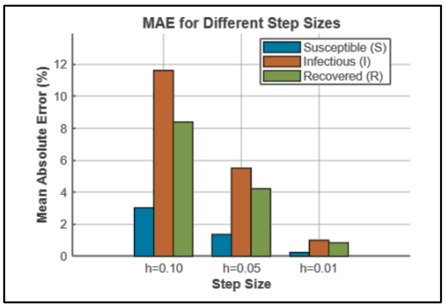


Fig. 4 Error Analysis

Table 3 MAE value

Step Size	MAE_S (%)	MAE_I (%)	MAE_R (%)
0.01	0.2374	0.9975	0.8385
0.05	1.3585	5.5059	4.2204
0.1	3.0316	11.6188	8.3934

The MAE percentage of susceptible, infected and recovered population are shown in Table 3. The findings indicate a consistent decline in MAE value as the step size decrease. This trend shows that numerical solution with smaller step size is closer to the reference solution and more accurate. the error analysis supported that the accuracy and reliability of the RK45 in solving the SIR model of HFMD in Sarawak.

3.4 Discussion

The RK45 proved effective in solving the SIR model of HFMD in Sarawak as the numerical simulation show. The simulated population dynamics exhibit the SIR model's expected theoretical characteristics. The error analysis also shows that as the step size is decreased, the MAE decreased significantly, indicating increased numerical accuracy. This demonstrates that the adaptive step size technique improves solution stability and dependability of the RK45 method when modelling HFMD transmission dynamics.

4. Conclusion

RK45 was successfully applied in this study for solving the SIR model of HFMD in Sarawak and numerical solutions were acquired via MATLAB. The simulation findings show that the RK45 produces stable and consistent numerical results. The population trends observed in the simulation results of the SIR model in this study follow the expected epidemiological behaviour of HFMD transmission. A study of different step sizes demonstrates that the numerical results are closer to the reference solution from the ode45 solver when the step size is smaller and these findings are supported by the error analysis where MAE values drop with decreasing step size.

This study's primary contribution is the comparison between the RK45 with multiple step sizes and the adaptive step ode45 solver in modelling HFMD transmission. This makes it easier to comprehend the effects of step size on the numerical solution's accuracy. But this study uses the simplified SIR model and does not include factors such as seasonal impacts, control measures and age groups. Additionally, the parameter values may not accurately reflect the current condition of HFMD in Sarawak since their values were derived from earlier research.

Future studies might extend the model to include more complex epidemic frameworks and use recent actual HFMD data to estimate the parameters. Alternative numerical solutions also can be investigated and compared in terms of accuracy and computational efficiency for solving the epidemiological model.

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Conflict of Interest

Authors declare that there is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Aideena Umairah Musa, Azila Md Sudin;* ***data collection:*** *Aideena Umairah Musa;* ***analysis and interpretation of results:*** *Aideena Umairah Musa;* ***draft manuscript preparation:*** *Aideena Umairah Musa, Azila Md Sudin. All authors reviewed and approved the final version of the manuscript.*

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