ENHANCED KNOWLEDGE IN SCIENCES AND TECHNOLOGY
e-ISSN: 2773-6385

EKST

Vol. 6 No. 1 (2026) 299-309
https://publisher.uthm.edu.my/periodicals/index.php/ekst

Solving System of Ordinary Differential Equation Using SAWI Transformation

Muhammad Adib Adenan[1], Noor Azliza Abd Latif[1]*, Norziha Che Him[1]

[1] *Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, UTHM Kampus Cawangan Pagoh, Hab Pendidikan Tinggi Pagoh, KM 1, Jalan Panchor, 84600 Pagoh, Muar, Johor, MALAYSIA.*

*Corresponding Author: azliza@uthm.edu.my
DOI: https://doi.org/10.30880/ekst.2026.06.01.036

Article Info

Received: 2 January 2026
Accepted: 20 January 2026
Available online: 9 July 2026

Keywords

SAWI Transformation, Ordinary Differential Equations, Integral Transform, Exact Solution, System of ODEs

Abstract

The study examines the use of Shifted and Weighted Integral (SAWI) transformation in the solution of systems of ordinary differential equations (ODEs). SAWI is a comparatively new integral transform that is based on Fourier integrals and it is characterized by its ability to translate a given differential equation into algebraic forms thus making the process of finding a solution a lot simpler and yet, preserving the accuracy of the solution. This paper uses the SAWI transformation method to dynamics of linear first-order and second-order ODEs with constant coefficients. This technique is proved to be efficient by its comparison with the precise solutions acquired by MATLAB. Numerical solutions of two representative problems such as a chemical kinetics model and a second-order system coupled system, the SAWI transformation yields a solution that is identical to the analytical solutions of the test problems involved. The results also propose the potential of SAWI as an effective, accurate and computationally feasible alternative to conventional numerical approaches to the solution of ODE systems with broad implications to applied mathematics, engineering, and scientific modelling.

1. Introduction

Ordinary differential equations (ODEs) are more than just theoretical concepts in math; ODEs are applied in many fields. In physics, which is in the domain of mechanics, ODEs present object motion under forces that include the second law of Newton. In biology, ODEs are used to model population growth as well as the spread of viruses. Also, in economics ODEs play a role in that of investment growth or decline over time. In engineering these equations are a tool for design of structures like electric circuits, bridges, and control systems [1],[2].

One of the earliest starting points for the study of ODEs serving as a preface to mathematics, engineering, and science students is the textbook by Morris Tenenbaum and Harry Pollard in 1963 [3]. Following these steps, further contributed to the topic by studying damping descriptions using fractional operators [4]. The crucial role played by numerical solutions. Numerical solutions come very handy for ODEs where exact or analytical solutions are not readily obtainable or cannot be given explicitly [5]. The SAWI transformation is an integral transformation derived from the Fourier integral. It was proposed as an effective tool for solving linear and nonlinear ODE and partial differential equations. The basic properties of the SAWI transformation include linearity, scaling, and translation, which make it versatile for different types of ODEs [6]. A major aspect of the SAWI transformation is that it can accommodate both initial conditions and derivatives, which are both fundamental to the dynamic behaviour of a system. The SAWI transformation also has linear and scalable attributes and therefore can be easily adapted to address a wide array of problems across all fields of engineering and science [1],[2].

Solving systems of ODEs is a difficult issue which also grows when many interrelated variables are present. The traditional approach and non-traditional methods of solution is time and computation heavy. Also, it is a fact that not all these methods are guaranteed to present exact solutions which is a break point in very sensitive areas like chemical engineering or aerospace. Therefore, there is a need for a method that can provide accurate solutions with less computational effort. The SAWI transformation presents a promising approach, but its application to systems of ODEs remains underexplored and less known among undergraduate researchers [1].

This study aims to solve systems of ODEs using the SAWI transformation method and compare SAWI transformation method with exact solution.

2. Methodology

This section outlines the systematic approach used to apply the SAWI transformation in solving systems of ODEs. The study focuses on linear systems with constant coefficients, and the procedure is implemented using mathematical software MATLAB R2025b for validation and comparison.

2.1 System of Ordinary Differential Equations

The standard form of system of ODEs of the n th order with conditions is considered as

$$\begin{gathered} f_1\left(x, y_1(x), y_1'(x), \cdots, y_1^{(n)}(x), y_2(x), y_2'(x), \cdots, y_2^{(n)}(x), y_\rho(x), y_\rho'(x), \cdots, y_\rho^{(n)}(x)\right) = 0 \\ f_2\left(x, y_1(x), y_1'(x), \cdots, y_1^{(n)}(x), y_2(x), y_2'(x), \cdots, y_2^{(n)}(x), y_\rho(x), y_\rho'(x), \cdots, y_\rho^{(n)}(x)\right) = 0 \\ \vdots \\ f_\rho\left(x, y_1(x), y_1'(x), \cdots, y_1^{(n)}(x), y_2(x), y_2'(x), \cdots, y_2^{(n)}(x), y_\rho(x), y_\rho'(x), \cdots, y_\rho^{(n)}(x)\right) = 0 \end{gathered} \tag{1}$$

with initial values given for

$$\begin{gathered} y_1(x_0) = y_{1_0}, y_1'(x_0) = y_{1_0}', \cdots, y_1^{(n-1)}(x_0) = y_{1_0}^{(n-1)} \\ y_2(x_0) = y_{2_0}, y_2'(x_0) = y_{2_0}', \cdots, y_2^{(n-1)}(x_0) = y_{2_0}^{(n-1)} \\ \vdots \\ y_\rho(x_0) = y_{\rho_0}, y_\rho'(x_0) = y_{\rho_0}', \cdots, y_\rho^{(n-1)}(x_0) = y_{\rho_0}^{(n-1)} \end{gathered} \tag{2}$$

Where f_1 for $i = 1,2, \dots, \rho$ are nonlinear continuous functions if its argument.

This study emphasizes the first order of a system of ODEs. The general form of first order of the system of ODEs are:

$$\begin{gathered} y_1' = f_1(x, y_1, y_2, \dots, y_\rho) \\ y_2' = f_2(x, y_1, y_2, \dots, y_\rho) \\ \vdots \\ y_\rho' = f_\rho(x, y_1, y_2, \dots, y_\rho) \end{gathered} \tag{3}$$

with initial conditions

$$y_1(x_0) = y_{1_0}, y_2(x_0) = y_{2_0}, \dots, y_\rho(x_0) = y_{\rho_0} \tag{4}$$

2.2 SAWI Transformation

The SAWI transforms of the function $\omega(t), y \geq 0$ is given by

$$S\{\omega(t)\} = \frac{1}{\sigma^2}\int_0^\infty \omega(t) e^{-\left(\frac{1}{\sigma}\right)t}\, dt = T(\sigma), \sigma > 0 \tag{5}$$

$$\begin{aligned} S\left(e^{kt}\omega(kt)\right) &= \frac{1}{\sigma^2}\int_0^\infty \omega(t) e^{-\left(\frac{1-k\sigma}{\sigma}\right)t}\, dt = T\left(\frac{\sigma}{1-k\sigma}\right) \\ &= \frac{1}{(1-k\sigma)^2}\left[\frac{(1-k\sigma)^2}{\sigma^2}\int_0^\infty \omega(t) e^{-\left(\frac{1-k\sigma}{\sigma}\right)t}\, dt\right] \\ &= \frac{1}{(1-k\sigma)^2} T\left(\frac{\sigma}{1-k\sigma}\right) \end{aligned} \tag{6}$$

2.2.1 SAWI transformation of derivatives of function

The following results are obtained by applying integration by parts to the SAWI transform and can be found in [2]. Table 3.1 presents the SAWI transformation of derivatives of function.

Table 1 SAWI transformation of derivatives of function

No.	Function of derivatives	SAWI transformation of derivatives
1.	$\omega'(t)$	$\frac{1}{\sigma}T(\sigma)-\frac{1}{\sigma^2}\omega(0)$
2.	$\omega''(t)$	$\frac{1}{\sigma^2}T(\sigma)-\frac{1}{\sigma^3}\omega(0)-\frac{1}{\sigma^2}\omega'(0)$
3.	$\vdots$	$\vdots$
4.	$\omega^{(\rho)}(t)$	$\frac{1}{\sigma^\rho}T(\sigma)-\sum_{k=0}^{\rho-1}\left(\frac{1}{\sigma}\right)^{\rho-(k-1)}\omega^{(k)}(0)$

2.2.2 Characteristics of SAWI transformation

The following are the characteristics of SAWI Transformation, which make it suitable for solving system of ODEs.

i. Linearity property of SAWI transformation

$$S[\alpha\omega_1(t)+\beta\omega_2(t)]=[\alpha T_1(\sigma)+\beta T_2(\sigma)]$$

Where α and β are arbitrary constants.

ii. Scaling property of SAWI transformation

$$S\{\omega(kt)\}=kT(k\sigma)$$

iii. Translation property of SAWI transformation

$$S\{e^{kt}\omega(kt)\}=\frac{1}{(1-k\sigma)^2}T\left(\frac{\sigma}{1-k\sigma}\right)$$

2.2.3 Some fundamental functions with their SAWI transformation

Some fundamental concepts of SAWI transformation involve its basic formula for transforming functions into algebraic expressions and the derivative formula, which allows differential terms to be simplified while automatically incorporating initial conditions [2].

Table 2 Fundamental functions with their SAWI transformation

No.	$\boldsymbol{\omega(t)}$	$\boldsymbol{S\{\omega(kt)\}=kT(k\sigma)}$
1.	1	$\frac{1}{\sigma}$
2.	t	1
3.	t^2	2σ
4.	$t^2,\rho\in N$	$\rho!\,\sigma^{\rho-1}$
5.	$t^\rho,\rho>-1$	$T(\rho+1)\sigma^{\rho-1}$
6.	e^{at}	$\frac{1}{\sigma(1-a\sigma)}$
7.	$\sin(at)$	$\frac{a}{1+(a\sigma)^2}$

Table 2 (Continue)

8.	$\cos(at)$	$\frac{1}{\sigma(1+(a\sigma)^2)}$
9.	$\sinh(at)$	$\frac{a}{1-(a\sigma)^2}$
10.	$\cosh(at)$	$\frac{1}{\sigma(1-(a\sigma)^2)}$

2.2.4 Inverse SAWI transformation of some fundamental functions

The function $\omega(t)$ is called the inverse SAWI transformation of the function $T(\sigma)$ if it has the following property $S\{\omega(t)\} = T(\sigma)$. In symbolic form, it is written as $S^{-1}\{T(\sigma)\} = \omega(t)$ [2].

Table 3 Inverse SAWI transformation of some fundamental functions

No.	$\boldsymbol{T(\sigma)}$	$\boldsymbol{S\{\omega(kt)\} = kT(k\sigma)}$
1.	$\frac{1}{\sigma}$	1
2.	1	t
3.	σ	$\frac{t^2}{2!}$
4.	$\sigma^{\rho-1}, \rho \in N$	$\frac{t^\rho}{\rho!}$
5.	$\sigma^{\rho-1}, \rho > -1$	$\frac{t^\rho}{T(\rho+1)}$
6.	$\frac{1}{\sigma(1-a\sigma)}$	e^{at}
7.	$\frac{a}{1+(a\sigma)^2}$	$\frac{\sin(at)}{a}$
8.	$\frac{1}{\sigma(1+(a\sigma)^2)}$	$\cos(at)$
9.	$\frac{a}{1-(a\sigma)^2}$	$\frac{\sinh(at)}{a}$
10.	$\frac{1}{\sigma(1-(a\sigma)^2)}$	$\cosh(at)$

The following table presents the list of essential symbols used throughout this study along with their corresponding meanings.

Table 4 List of Symbol

List of Symbol	Meaning
t	Independent (time) variable
$y_i(t)$	Dependent / state variables of the system
$y_i'(t)$	First derivative of y_i with respect to t
n	Order of the differential equation
ρ	Number of equations in the system
x_0	Initial value of independent variable
y_{i0}	Initial value of y_i
$S\{\cdot\}$	SAWI transformation operator
$S^{-1}\{\cdot\}$	Inverse SAWI transformation
$\omega(t)$	General function in SAWI transformation
σ	SAWI transform parameter ($\sigma > 0$)
$T(\sigma)$	SAWI transform of $\omega(t)$
p_{ij}	Constant coefficients of the system
$q_i(t)$	Source / input functions
r_i	Initial condition values
Y	State vector
P	Coefficient matrix
Q(t)	Input vector
R	Initial condition vector
k_1, k_2	Reaction rate constants
β	Initial concentration constant

2.2.5 SAWI transformation for system of ODEs

Consider the following first order system of ODEs

$$\frac{dy_1}{dt} = p_{11}y_1 + p_{12}y_2 + \cdots + p_{1\rho}y_\rho + q_1(t)$$
$$\frac{dy_2}{dt} = p_{21}y_1 + p_{22}y_2 + \cdots + p_{2\rho}y_\rho + q_2(t)$$
$$\vdots$$
$$\frac{dy_\rho}{dt} = p_{\rho 1}y_1 + p_{\rho 2}y_2 + \cdots + p_{\rho\rho}y_\rho + q_\rho(t) \tag{7}$$

with initial conditions

$$y_1(0) = r_1, y_2(0) = r_2, \cdots, y_\rho(0) = r_\rho \tag{8}$$

where:

$$p_{11}, p_{12}, \cdots, p_{1\rho},$$
$$p_{21}, p_{22}, \cdots, p_{2\rho},$$
$$\vdots$$
$$p_{\rho 1}, p_{\rho 2}, \cdots, p_{\rho\rho},$$

The matrix visualization of the system (7) with initial condition (8) is

$$\frac{dY}{dt} = PY + Q(t) \tag{9}$$

with

$$Y(0) = R$$

where

$$\frac{dY}{dt} = \begin{bmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \\ \vdots \\ \frac{dy_\rho}{dt} \end{bmatrix}, p = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1\rho} \\ p_{21} & p_{22} & \cdots & p_{2\rho} \\ \vdots & \vdots & \cdots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{\rho\rho} \end{bmatrix}, Q = \begin{bmatrix} q_{1(t)} \\ q_{2(t)} \\ \vdots \\ q_{\rho(t)} \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_\rho \end{bmatrix}, Y(0) = \begin{bmatrix} y_1(0) \\ y_2(0) \\ \vdots \\ y_\rho(0) \end{bmatrix}, and\ R = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_\rho \end{bmatrix}$$

Operating SAWI transforms on system (7) and applying initial conditions (8), we have

$$\begin{aligned} \left(\tfrac{1}{\sigma} - p_{11}\right) S\{y_1\} - p_{12} S\{y_2\} - \cdots - p_{1\rho} S\{y_\rho\} &= S\{q_1(t)\} + \tfrac{r_1}{\sigma^2} \\ -p_{21} S\{y_1\} + \left(\tfrac{1}{\rho} - p_{22}\right) S\{y_2\} - \cdots - p_{2\rho} S\{y_\rho\} &= S\{q_2(t)\} + \tfrac{r_2}{\sigma^2} \\ &\vdots \\ -p_{\rho 1} S\{y_1\} - p_{\rho 2} S\{y_2\} - \cdots + \left(\tfrac{1}{\sigma} - p_{\rho\rho}\right) S\{y_\rho\} &= S\{q_\rho(t)\} + \tfrac{r_\rho}{\sigma^2} \end{aligned} \tag{10}$$

Using Cramer rule, the solutions of algebraic system (10) are given by

$$S\{y_1\} = \frac{\begin{vmatrix} S\{q_1(t)\} + \frac{r_1}{\sigma^2} & -p_{12} & \cdots & -p_{1\rho} \\ S\{q_2(t)\} + \frac{r_2}{\sigma^2} & \left(\frac{1}{\rho} - p_{22}\right) & \cdots & -p_{2\rho} \\ \vdots & \vdots & \cdots & \vdots \\ S\{q_\rho(t)\} + \frac{r_\rho}{\sigma^2} & -p_{\rho 2} & \cdots & \left(\frac{1}{\sigma} - p_{\rho\rho}\right) \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{\sigma} - p_{11}\right) & -p_{12} & \cdots & -p_{1\rho} \\ -p_{21} & \left(\frac{1}{\rho} - p_{22}\right) & \cdots & -p_{2\rho} \\ \vdots & \vdots & \cdots & \vdots \\ -p_{\rho 1} & -p_{\rho 2} & \cdots & \left(\frac{1}{\sigma} - p_{\rho\rho}\right) \end{vmatrix}}$$

$$S\{y_2\} = \frac{\begin{vmatrix} \left(\frac{1}{\sigma} - p_{11}\right) & S\{q_1(t)\} + \frac{r_1}{\sigma^2} & \cdots & -p_{1\rho} \\ -p_{21} & S\{q_2(t)\} + \frac{r_2}{\sigma^2} & \cdots & -p_{2\rho} \\ \vdots & \vdots & \cdots & \vdots \\ -p_{\rho 1} & S\{q_\rho(t)\} + \frac{r_\rho}{\sigma^2} & \cdots & \left(\frac{1}{\sigma} - p_{\rho\rho}\right) \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{\sigma} - p_{11}\right) & -p_{12} & \cdots & -p_{1\rho} \\ -p_{21} & \left(\frac{1}{\rho} - p_{22}\right) & \cdots & -p_{2\rho} \\ \vdots & \vdots & \cdots & \vdots \\ -p_{\rho 1} & -p_{\rho 2} & \cdots & \left(\frac{1}{\sigma} - p_{\rho\rho}\right) \end{vmatrix}}$$

$$S\{y_\rho\} = \begin{vmatrix} \left(\frac{1}{\sigma} - p_{11}\right) & -p_{1\rho} & \cdots & S\{q_1(t)\} + \frac{r_1}{\sigma^2} \end{vmatrix}$$

$$\begin{array}{|cccc|}
-p_{21} & \left(\frac{1}{\sigma}-p_{22}\right) & \cdots & S\{q_2(t)\}+\frac{r_2}{\sigma^2} \\
\vdots & \vdots & \cdots & \vdots \\
-p_{\rho 1} & -p_{\rho 2} & \cdots & S\{q_\rho(t)\}+\frac{r_\rho}{\sigma^2} \\
\hline
\left(\frac{1}{\sigma}-p_{11}\right) & -p_{12} & \cdots & -p_{1\rho} \\
-p_{21} & \left(\frac{1}{\sigma}-p_{22}\right) & \cdots & -p_{2\rho} \\
\vdots & \vdots & \cdots & \vdots \\
-p_{\rho 1} & -p_{\rho 2} & \cdots & \left(\frac{1}{\sigma}-p_{\rho\rho}\right)
\end{array} \tag{11}$$

Expanding these determinants, we have

$$S\{y_1\}, S\{y_2\}, \cdots, S\{y_\rho\} \tag{12}$$

The inverse SAWI transformation of these functions gives the required values of

$$y_1, y_2, \cdots, y_\rho \tag{13}$$

3. Result and Discussion

This section presents the numerical and graphical outcomes obtained from solving systems of ordinary differential equations using the SAWI transformation method. The results are compared with exact solutions derived analytically and computed using MATLAB.

3.1 Test Problem: First-Order Chemical Kinetics Model

Consider the following system of ordinary differential equations from the application of the field physical-chemical for determining the concentrations y_1, y_2 and y_3 of two reactants $k_1$and k_2 [1].

$$\begin{aligned} y_1' &= -k_1 y_1 \\ y_2' &= k_1 y_1 - k_2 y_2 \\ y_3' &= k_2 y_2 \end{aligned}$$

with initial condition

$$y_1(0)=\beta, y_2(0)=0, y_3(0)=0$$

and given the exact solution (from MATLAB R2025b),

$$\begin{aligned} y_1(t) &= \beta e^{-k_1 t} \\ y_2(t) &= \frac{\beta k_1 e^{-k_2 t}}{-k_2+k_1} - \frac{\beta k_1 e^{-k_1 t}}{-k_2+k_1} \\ y_3(t) &= \beta - \frac{\beta k_1 e^{-k_2 t}}{-k_2+k_1} + \frac{\beta k_2 e^{-k_1 t}}{-k_2+k_1} \end{aligned} \tag{14}$$

When we use the value of the constant $\beta = 1, k_1 = 1$ and $k_2 = 2$, the system becomes

$$\begin{aligned} y_1' &= -y_1 \\ y_2' &= y_1 - 2y_2 \\ y_3' &= 2y_2 \end{aligned} \tag{15}$$

with

$$y_1(0)=1, y_2(0)=0, y_3(0)=0 \tag{16}$$

And given the exact solution for $\beta = 1, k_1 = 1$ and $k_2 = 2$ (from MATLAB R2025b)

$$\begin{aligned} y_1(t) &= e^{-t} \\ y_2(t) &= e^{-t} - e^{-2t} \\ y_3(t) &= 1 + e^{-2t} - 2e^{-t} \end{aligned} \tag{17}$$

3.1.1 SAWI transformation Solution

The solution of the test problem using SAWI transformation as follows: [1]
Applying the SAWI transformation method to the system (15), we get

$$\begin{aligned} S\{y_1'\} + S\{y_1\} &= S\{0\} \\ -S\{y_1\} + 2S\{y_2\} + S\{y_2'\} &= S\{0\} \end{aligned}$$

$$-2S\{y_2\} + S\{y_3'\} \qquad = S\{0\} \tag{18}$$

Using the derivative function of SAWI transformation in (18) and substitute initial condition (16), we have

$$\begin{aligned} \left(\frac{1}{\sigma}+1\right)S\{y_1\} &= \frac{1}{\sigma^2} \\ -S\{y_1\} + \left(\frac{1}{\sigma}+2\right)S\{y_2\} &= 0 \\ -2S\{y_2\} + \left(\frac{1}{\sigma}\right)S\{y_3\} &= 0 \end{aligned} \tag{19}$$

Using Cramer rule, the primitives of (18) are determined by

$$S\{y_1\} = \frac{\begin{vmatrix} \frac{1}{\sigma^2} & 0 & 0 \\ 0 & \left(\frac{1}{\sigma}+2\right) & 0 \\ 0 & -2 & \frac{1}{\sigma} \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{\sigma}+1\right) & 0 & 0 \\ -1 & \left(\frac{1}{\sigma}+2\right) & 0 \\ 0 & -2 & \frac{1}{\sigma} \end{vmatrix}} = \frac{1}{\sigma(1+\sigma)} \tag{20}$$

$$S\{y_2\} = \frac{\begin{vmatrix} \left(\frac{1}{\sigma}+1\right) & \frac{1}{\sigma^2} & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sigma} \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{\sigma}+1\right) & 0 & 0 \\ -1 & \left(\frac{1}{\sigma}+2\right) & 0 \\ 0 & -2 & \frac{1}{\sigma} \end{vmatrix}} = \frac{1}{\sigma(1+\sigma)} - \frac{1}{\sigma(1+2\sigma)} \tag{21}$$

$$S\{y_3\} = \frac{\begin{vmatrix} \left(\frac{1}{\sigma}+1\right) & 0 & \frac{1}{\sigma^2} \\ -1 & \left(\frac{1}{\sigma}+2\right) & 0 \\ 0 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{\sigma}+1\right) & 0 & 0 \\ -1 & \left(\frac{1}{\sigma}+2\right) & 0 \\ 0 & -2 & \frac{1}{\sigma} \end{vmatrix}} = \frac{1}{\sigma}\left(1 - \frac{2}{1+\sigma} + \frac{1}{1+2\sigma}\right) \tag{22}$$

Operating inverse SAWI transform on equations (19), (20) and (21), we have

$$y_1 = S^{-1}\left\{\frac{1}{\sigma(1+\sigma)}\right\} = e^{-t} \tag{23}$$

$$y_2 = S^{-1}\left\{\frac{1}{\sigma(1+\sigma)} - \frac{1}{\sigma(1+2\sigma)}\right\} = e^{-t} - e^{-2t} \tag{24}$$

$$y_3 = S^{-1}\left\{\frac{1}{\sigma}\left(1 - \frac{2}{1+\sigma} + \frac{1}{1+2\sigma}\right)\right\} = 1 - 2e^{-t} + e^{-2t} \tag{25}$$

This shows that the SAWI transformation method can be used to solve systems of ODEs accurately. Since the results are identical to the exact solutions, the SAWI method is a reliable and suitable alternative when exact solutions are available, as it gives the same answers without approximation.

3.1.2 Tabulated Results for SAWI Transformation

Using MATLAB 2025b, we solve the system (14) with the initial condition (15) to obtain the exact solution. The numerical results obtained from exact values are presented in Tables 4.1 to 4.3 are compared with the SAWI transformation. Tables 5, 6 and 7 show the result of $y1(t)$, $y2(t)$ and $y3(t)$ for β = 1, $k1$ = 1 and $k2$ = 2.

Table 5 Comparison of $\boldsymbol{y_1(t)}$ between SAWI transformation and exact solution

t	Exact Value	SAWI	|Exact − SAWI|
0	1.0000	1.0000	0.0000
1	0.3679	0.3679	0.0000
2	0.1353	0.1353	0.0000
3	0.0498	0.0498	0.0000
4	0.0183	0.0183	0.0000
5	0.0067	0.0067	0.0000
6	0.0025	0.0025	0.0000

Table 6 Comparison of $\boldsymbol{y_2(t)}$ between SAWI transformation and exact solution

t	Exact Value	SAWI	|Exact − SAWI|
0	0.0000	0.0000	0.0000
1	0.2325	0.2325	0.0000
2	0.1170	0.1170	0.0000
3	0.0473	0.0473	0.0000
4	0.0180	0.0180	0.0000
5	0.0067	0.0067	0.0000
6	0.0025	0.0025	0.0000

Table 7 Comparison of $\boldsymbol{y_3(t)}$ between SAWI transformation and exact solution

t	Exact Value	SAWI	|Exact − SAWI|
0	0.0000	0.0000	0.0000
1	0.3996	0.3996	0.0000
2	0.7476	0.7476	0.0000
3	0.9029	0.9029	0.0000
4	0.9637	0.9637	0.0000
5	0.9866	0.9866	0.0000
6	0.9950	0.9950	0.0000

From Tables 5, 6, and 7, it can be observed that the SAWI transformation results are identical to the exact solution for all values of t from 0 to 6. The absolute error is zero for every entry, confirming that the SAWI method provides exact solutions for this system.

3.1.3 Graphical Representation of Results for SAWI Transformation

Fig. 1 to Fig. 3 show the comparison results of $y_1(t)$, $y_2(t)$ and $y_3(t)$ with the exact solution for $\beta = 1, k_1 = 1$ and $k_2 = 2$.

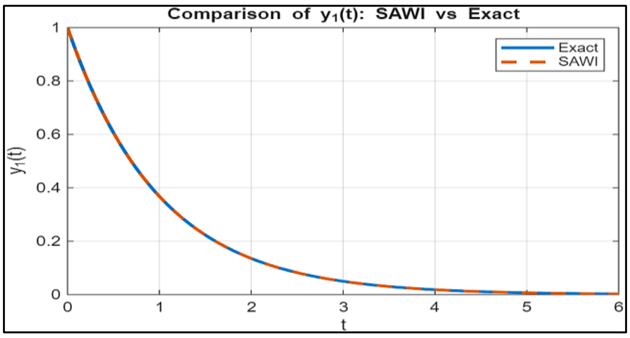

Fig. 1 Comparison of $\boldsymbol{y_1}$obtained using the SAWI transformation with the exact solution

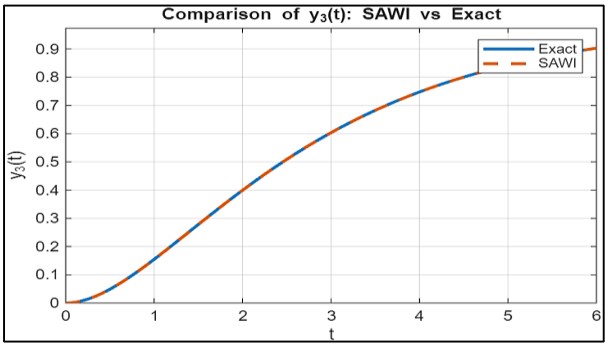

Fig. 2 Comparison of $\boldsymbol{y_2}$obtained using the SAWI transformation with the exact solution

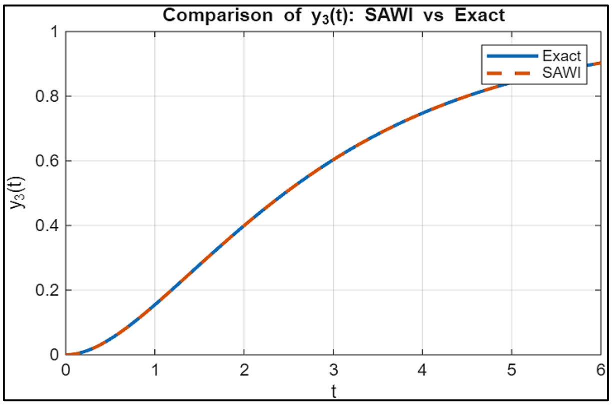

Fig. 3 Comparison of $\boldsymbol{y_3}$ obtained using the SAWI transformation with the exact solution

4. Discussion

The results confirm that the SAWI transformation method provides exact solutions for the first-order linear ODE system modelling chemical kinetics. SAWI's strength lies in its algebraic transform approach, which converts differential equations into a solvable algebraic system, eliminating discretization errors and avoiding stability issues common in iterative methods like Runge–Kutta. The method also automatically incorporates initial conditions during transformation, simplifying the solution process.

These findings support earlier studies by [1] and [2], reinforcing SAWI's reliability as an exact and efficient method for linear ODE systems. SAWI is particularly suitable where analytical precision is required, offering a viable alternative to conventional numerical techniques in both educational and applied settings.

5. Conclusion

The SAWI transformation method was employed in this study to successfully solve selected systems of ordinary differential equations. Based on the results obtained through the application of the SAWI transformation and the subsequent analysis of the data, the derived analytical solutions were validated against appropriate exact solutions. The results and graphical comparisons presented demonstrate that the solutions obtained using the SAWI transformation exhibit excellent agreement with the exact solutions across all examples considered. The close similarity of the solution curves indicates that the discrepancy between the SAWI solutions and the exact solutions is minimal and negligible. A notable advantage of the SAWI transformation is its simplicity of implementation, reduced computational complexity, and its capability to solve ODE systems without the need for discretization or linearization. The outcomes of this study confirm that the SAWI transformation serves as an efficient and accurate analytical method for solving systems of ordinary differential equations.

Despite such positive findings, some limitations of the study are to be considered. The study is also restricted to a small range of examples with certain initial conditions, which are not necessarily representative of the behaviour of all the ODE systems. Furthermore, the examples utilized possess known exact solutions; thus, the performance of the SAWI transformation in scenarios lacking exact solutions remains unexplored. Also, the analysis of the relatively simple ODE systems has been done and more complex or higher-order systems are not addressed. In light of these shortcomings and to elaborate on the existing results, some suggestions on the future research are discussed. Future effort can be used to apply the SAWI transformation to nonlinear systems and uses of nonlinear ODEs as well as real world engineering problems with variable coefficients. A comparative analysis between the SAWI transformation and established numerical methods, such as Runge–Kutta techniques, could further elucidate its accuracy and efficiency. Lastly, the SAWI transformation holds potential for extension to other classes of differential equations and for application to a broader range of practical problems in science and engineering.

Acknowledgement

The authors would thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

Conflict of Interest

Authors declare that there is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design****: Adib Adenan, Noor Azliza Abd Latif;* ***data collection****: Adib Adenan;* ***analysis and interpretation of results****: Adib Adenan, Noor Azliza Abd Latif;* ***draft manuscript preparation****: Adib Adenan, Noor Azliza Abd Latif, Norziha Che Him. All authors reviewed the results and approved the final version of the manuscript*

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