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Comparison of Fourth-Order Runge-Kutta and Second-Order Taylor Series Method for Solving Influenza Cases

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Abstract

This research investigates the modelling of influenza disease transmission using Fourth-Order Runge-Kutta method and Second-Order Taylor Series method. Influenza is categorized as an acute contagious viral respiratory disease that is usually characterized by the presence of fever, cough, sore throat, headache, and myalgia. It also a viral respiratory infection caused by viruses that spread rapidly and has an enormous effect on public health systems throughout the world, including Malaysia. Second-Order Taylor Series method utilized in this study because it enhances the basic technique such as Euler Method by using both the first and second derivatives of a function in its calculation. The Fourth-Order Runge-Kutta method is also utilized in this study to make a comparison with Second-Order Taylor Series method because the RK4 method utilises four different slopes and uses both first and second derivatives at the beginning of each step to calculate its next value. Mean Absolute Error (MAE) is then used in this study to decide which one is more reliable or accurate. The model was executed using MATLAB software, which produces numerical solutions that simulate the dynamics of influenza disease transmission. At the end of the study, the result shows that MAE for Runge-Kutta Fourth Order Method got 57.7458 while MAE for Second-Order Taylor Series Method got 57.8553. It shows that RK4 method got lower value of MAE than Second-Order Taylor Series method.

1. Introduction

Mathematical models have been used extensively for the understanding of transmission dynamics and the evaluation of control measures for such diseases. Among them, the SEIR model-that segments the population into Susceptible (S), Exposed (E), Infectious (I), and Recovered (R) classes-has proven to be one of the most common models, for the latent or incubation disease case such as influenza, COVID-19, and measles [1]. The SEIR model has a key role to play in estimating the basic reproduction number (R_0), evaluating the effectiveness of intervention strategies (e.g., vaccination, quarantine), and forecasting the long-term course of an epidemic. To study the SEIR model, a nonlinear system of ordinary differential equations (ODEs) needs to be solved. But in most cases, analytical solutions are impractical because real-world parameters and interventions are too complex. This makes it necessary to use numerical methods, which provide approximations at discrete time points. Two of the popular numerical methods for ODE solving are the Second Order Taylor Series method and the Fourth-Order Runge-Kutta (RK4) method. In contrast, the RK4 method is a more advanced method that calculates intermediate points within each time step to attain greater accuracy and reduced numerical error.

Even though it takes more time to calculate it using MATLAB, RK4 is generally more accurate and stable and thus is usually the preferred method when modelling sensitive systems like disease outbreaks. The Second Order Taylor Series method offers an improved alternative for first-order method such as Euler method by involve both the first and second derivatives in its approximation. This method achieves a good balance between accuracy and computational cost [2]. Despite these method's advantages, limited study has compared the performance of RK4 and Second-Order Taylor Series method in solving the SEIR model, particularly in the context of influenza modelling in Malaysia. To fill this gap, this research aims to compare and evaluate the performance of Fourth-Order Runge-Kutta (RK4) method and the Second-Order Taylor Series method to solve SEIR model for influenza transmission. Both methods will be applied using MATLAB to simulate the spread of the disease and evaluate their accuracy and computational efficiency. Furthermore, the simulation results will be compared with actual data to determine how well each method captures actual outbreak trends. Based on the previous research, [3] monthly influenza case data from Malaysia from 2018 for a year will be used for this purpose.

This research aim to solve the SEIR model of influenza cases with Second Order Taylor Series method and the Fourth-Order Runge-Kutta method, to compare the accuracy of both numerical methods by measuring their errors against the real 2018 influenza case data using MATLAB 2024a and the last one is to evaluate the numerical solutions produced by both methods over time, that focusing on their error values and solution trends.

2. Methodology

In this chapter, more detail will be explained regarding the Influenza disease model and Second Order Taylor Series method. This chapter involves three subtopics which are Influenza disease model, Fourth-Order Runge-Kutta (RK4) method and Second-Order Taylor Series method.

2.1 Influenza Disease Model (Basic SEIR Model)

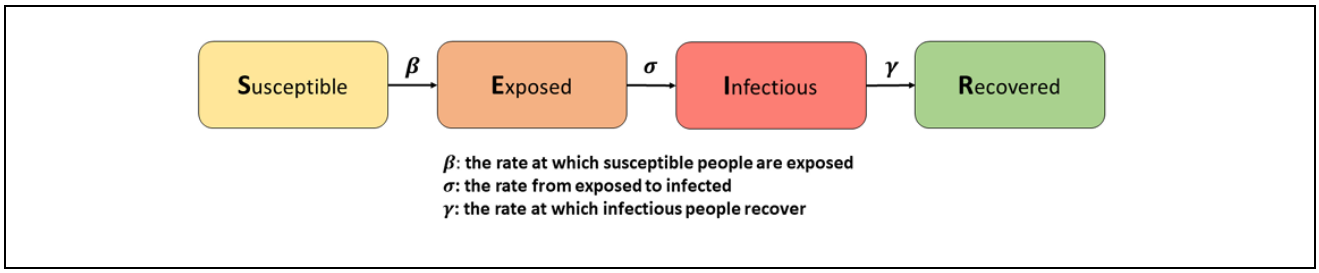

Fig. 1 SEIR Model diagram [4]

The SEIR model is a compartmental model that used to simulate the spread of infectious diseases, split the population into four groups which is S (Susceptible) for a people who have chances to het the disease, E (Exposed) for a people who are not yet infectious but have been exposed to the diseases, I (Infected) for a people who is the carrier of the disease and have the capability to infect others and the last one is R (Recovered) for the people who have recovered and can no longer infect the others.

Table 1 Parameter of SEIR model (Basic SEIR model) [6], [7], [8], [9]

Symbol	Meaning
S	Susceptible
E	Exposed
I	Infected
R	Recovered
β	Transmission rate.
σ	1/ Incubation period
γ	1/ Infectious period
N	Total population $= S + E + I + R$
S_0	Initial susceptible

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E_0	Initial exposed.
I_0	Initial infected.
R_0	Initial Recovered

The transitions between compartments are governed by a system of ordinary differential equations (ODEs) as follows.

$$\begin{aligned} \frac{dS}{dt} &= -\beta \frac{SI}{N} \\ \frac{dE}{dt} &= \beta \frac{SI}{N} - \sigma E \\ \frac{dI}{dt} &= \sigma E - \gamma I \\ \frac{dR}{dt} &= \gamma I \end{aligned} \tag{1}$$

This SEIR model assumes a closed population with no births, deaths or migration. This research only focusing on comparing numerical methods. Since the objective is to compare RK4 and Second-Order Taylor Series method, so basic SEIR model without demographics is ideal, sufficient, and appropriate. It helps the model stay simple and focused. Standard epidemiological definitions and values from previous research were used to determine the SEIR model parameters. The entire population size was determined to be 100,000 which is commonly used in epidemiological simulations to simplify analysis for modelling purposes. For influenza-like disease, the transmission rate β, represent the effective contact rate between susceptible and infected individuals, is usually assumed to be within a biologically reasonable range [5], [6]. The inverse of the incubation period, which is normally short for influenza infections [7]. Regarding the acute nature of influenza diseases, the recovery rate γ was determined as the inverse of the infectious period [8]. The initial number of infected individuals was set at 55 based on reported influenza cases in January 2018 [3]. Since the objective of the study is to evaluate numerical method rather than perform parameter estimation, the parameters were picked within biologically reasonable ranges reported in the literature.

2.2 Fourth-Order Runge-Kutta (RK4) Method

The RK4 method is a one-step method that utilise data from the earlier point to obtain y_{n+1}. RK4 can be utilised to calculate the previous value while figuring out the explicit four-step multistep method because it can offer more accurate values as opposed to alternative approaches.

RK4 method is the most accurate method compared to the other RK methods. Therefore, the RK4 is given by

$$y_{n+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{2}$$

where,

$$\begin{aligned} k_1 &= hf(x_i, y_i) \\ k_2 &= hf\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1\right) \\ k_3 &= hf\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2\right) \\ k_4 &= hf\left(x_i + h, y_i + k_3\right) \end{aligned} \tag{3}$$

2.3 Second-Order Taylor Series Method

This study uses the Second-Order Taylor Series method, which offers more accuracy than basic methods like Euler's method to solve the SEIR model numerically. This method works by including both the first and second derivatives in each step, allowing it to estimate the subsequent value accurately. It is suitable for disease modelling and improves the approximation of nonlinear systems, such as the SEIR model [2].

For a general differential equation,

$$\frac{dy}{dt} = f\left(t, y\right), \tag{4}$$

The Euler approximation is

$$y_{n+1} = y_n + hf\left(t_n, y_n\right) + \frac{h^2}{2} f'\left(t_n, y_n\right) \tag{5}$$

Table 2 Second-Order Taylor Series Method Parameter

Symbol	Meaning
y_n	Current value
h	Time step size
$f\left(t_n, y_n\right)$	First derivative
$f'\left(t_n, y_n\right)$	Second derivative with respect of time

2.4 Mean Absolute Error (MAE)

The Mean Absolute Error (MAE) is used to measure the accuracy of numerical solutions given by the Second-Order Taylor Series method and RK4 method. Without taking note of the direction of the errors, MAE measures the average absolute difference between the reported influenza case data and the predicted values from the SEIR model. MAE is usually used in numerical analysis and epidemiological modelling research because of it easy to use and clear interpretation [9], [10].

Mean Absolute Error is defined as:

$$MAE = \frac{1}{n}\sum_{i=0}^{n}\left|I_{\mathrm{mod}el,i} - I_{actual,i}\right| \tag{6}$$

where n is the total number of observation points, $I_{\mathrm{mod}el,i}$ reflect the number of infected individuals predicted by the SEIR model at time i, and $I_{actual,i}$ indicates the corresponding reported influenza cases. Higher numerical accuracy appears in a smaller MAE value, indicates a better agreement between the model output and real data.

2.5 Implementation of Fourth-Order Runge-Kutta Method

Numerical method of Influenza Cases model using RK4 method is given by

$$\begin{aligned}
f_w\left(t, S, I\right) &= \frac{dS(t)}{dt} = -\beta\frac{S(t)I(t)}{N} \\
f_w\left(t, S, E, I\right) &= \frac{dE(t)}{dt} = \beta\frac{S(t)I(t)}{N} - \sigma E(t) \\
f_x\left(t, E, I\right) &= \frac{dI(t)}{dt} = \sigma E(t) - \gamma I(t) \\
f_z\left(t, I\right) &= \frac{dR(t)}{dt} = \gamma I(t)
\end{aligned} \tag{7}$$

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The coefficients of the iteration, i are given as below:

$$
\begin{aligned}
w_1 &= h \cdot f_w\left(t_i, S_i, I_i\right) = h \cdot \left(-\beta \frac{S_i I_i}{N}\right) \\
x_1 &= h \cdot f_x\left(t_i, S_i, E_i, I_i\right) = h \cdot \left(\beta \frac{S_i I_i}{N} - \sigma E_i\right) \\
y_1 &= h \cdot f_y\left(t_i, E_i, I_i\right) = h \cdot \left(\sigma E_i - \gamma I_i\right) \\
z_1 &= h \cdot f_z\left(t_i, I_i\right) = h \cdot \left(\gamma I_i\right)
\end{aligned} \tag{8}
$$

$$
w_2 = h \cdot f_w\left(t_i + \frac{h}{2}, S_i + \frac{w_1}{2}, I_i + \frac{y_1}{2}\right) = h \cdot \left[-\beta \frac{\left(S_i + \frac{w_1}{2}\right)\left(I_i + \frac{y_1}{2}\right)}{N}\right]
$$

$$
x_2 = h \cdot f_x\left(t_i + \frac{h}{2}, S_i + \frac{w_1}{2}, E_i + \frac{x_1}{2}, I_i + \frac{y_1}{2}\right) = h \cdot \left[\beta \frac{\left(S_i + \frac{w_1}{2}\right)\left(I_i + \frac{y_1}{2}\right)}{N} - \sigma\left(E_i + \frac{x_1}{2}\right)\right] \tag{9}
$$

$$
y_2 = h \cdot f_y\left(t_i + \frac{h}{2}, E_i + \frac{x_1}{2}, I_i + \frac{y_1}{2}\right) = h \cdot \left[\sigma\left(E_i + \frac{x_1}{2}\right) - \gamma\left(I_i + \frac{y_1}{2}\right)\right]
$$

$$
z_2 = h \cdot f_z\left(t_i + \frac{h}{2}, I_i + \frac{y_1}{2}\right) = h \cdot \left[\gamma\left(I_i + \frac{y_1}{2}\right)\right]
$$

$$
w_3 = h \cdot f_w\left(t_i + \frac{h}{2}, S_i + \frac{w_2}{2}, I_i + \frac{y_2}{2}\right) = h \cdot \left[-\beta \frac{\left(S_i + \frac{w_2}{2}\right)\left(I_i + \frac{y_2}{2}\right)}{N}\right]
$$

$$
x_3 = h \cdot f_x\left(t_i + \frac{h}{2}, S_i + \frac{w_2}{2}, E_i + \frac{x_2}{2}, I_i + \frac{y_2}{2}\right) = h \cdot \left[\beta \frac{\left(S_i + \frac{w_2}{2}\right)\left(I_i + \frac{y_2}{2}\right)}{N} - \sigma\left(E_i + \frac{x_2}{2}\right)\right] \tag{10}
$$

$$
y_3 = h \cdot f_y\left(t_i + \frac{h}{2}, E_i + \frac{x_2}{2}, I_i + \frac{y_2}{2}\right) = h \cdot \left[\sigma\left(E_i + \frac{x_2}{2}\right) - \gamma\left(I_i + \frac{y_2}{2}\right)\right]
$$

$$
z_3 = h \cdot f_z\left(t_i + \frac{h}{2}, I_i + \frac{y_2}{2}\right) = h \cdot \left[\gamma\left(I_i + \frac{y_2}{2}\right)\right]
$$

$$
\begin{aligned}
w_4 &= h\cdot f_w\left(t_i+h, S_i+w_3, I_i+y_3\right) = h\cdot\left[-\beta\frac{\left(S_i+w_3\right)\left(I_i+y_3\right)}{N}\right]\\
x_4 &= h\cdot f_x\left(t_i+h, S_i+w_3, E_i+x_3, I_i+y_3\right) = h\cdot\left[\beta\frac{\left(S_i+w_3\right)\left(I_i+y_3\right)}{N}-\sigma\left(E_i+x_3\right)\right]\\
y_4 &= h\cdot f_y\left(t_i+h, E_i+x_3, I_i+y_3\right) = h\cdot\left[\sigma\left(E_i+x_3\right)-\gamma\left(I_i+y_3\right)\right]\\
z_4 &= h\cdot f_z\left(t_i+h, I_i+y_3\right) = h\cdot\left[\gamma\left(I_i+y_3\right)\right]
\end{aligned}
\tag{11}
$$

Thus, the numerical solution of Influenza cases is:

$$
\begin{aligned}
S_{i+1} &= S_i+\frac{1}{6}\left(w_1+2w_2+2w_3+w_4\right)\\
E_{i+1} &= E_i+\frac{1}{6}\left(x_1+2x_2+2x_3+x_4\right)\\
I_{i+1} &= I_i+\frac{1}{6}\left(y_1+2y_2+2y_3+y_4\right)\\
R_{i+1} &= R_i+\frac{1}{6}\left(z_1+2z_2+2z_3+z_4\right)
\end{aligned}
\tag{12}
$$

Where h is the step size

2.6 Implementation of Second-Order Taylor Series Method into the Model

The initial values for the Susceptible (S), Exposed (E), Infectious (I), and Recovered (R) compartments are determined at the beginning of the simulation. The SEIR model equations are used to evaluate the first derivatives of each compartment using these initial values. The next values of the S, E, I and R are then calculated by using the Second-Order Taylor Series method, using both the first and second derivatives at the current time step. To obtain the numerical solutions of the SEIR model for the period of the simulation, this process is repeated iteratively.

$$
\begin{aligned}
S_{n+1} &= S_n+h\cdot f_s\left(t_n, S_n, E_n, I_n, R_n\right)+\frac{h^2}{2}\cdot f_s'\left(t_n, S_n, E_n, I_n, R_n\right)\\
E_{n+1} &= E_n+h\cdot f_E\left(t_n, S_n, E_n, I_n, R_n\right)+\frac{h^2}{2}\cdot f_E'\left(t_n, S_n, E_n, I_n, R_n\right)\\
I_{n+1} &= I_n+h\cdot f_I\left(t_n, S_n, E_n, I_n, R_n\right)+\frac{h^2}{2}\cdot f_I'\left(t_n, S_n, E_n, I_n, R_n\right)\\
R_{n+1} &= R_n+h\cdot f_R\left(t_n, S_n, E_n, I_n, R_n\right)+\frac{h^2}{2}\cdot f_R'\left(t_n, S_n, E_n, I_n, R_n\right)
\end{aligned}
\tag{13}
$$

Current derivatives:

$$
f_S = -\frac{\beta SI}{N},\quad f_E = \frac{\beta SI}{N}-\sigma E,\quad f_I = \sigma E-\gamma I,\quad f_R = \gamma I \tag{14}
$$

For S:

$$
\frac{df_S}{dt} = -\frac{\beta}{N}\left(I\frac{dS}{dt}+S\frac{dI}{dt}\right) = -\frac{\beta}{N}\left(If_S+Sf_I\right) \tag{15}
$$

Substitute f_S and f_I:

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$$\frac{df_S}{dt} = -\frac{\beta}{N}\left[I\left(-\frac{\beta SI}{N} \right) + S\left(\sigma E - \gamma I \right) \right] = \frac{\beta^2 SI^2}{N^2} - \frac{\beta S}{N}\left(\sigma E - \gamma I \right) \tag{16}$$

For E :

$$\frac{df_E}{dt} = \frac{\beta}{N}\left(I\frac{dS}{dt} + S\frac{dI}{dt} \right) - \sigma\frac{dE}{dt} = \frac{\beta}{N}\left(If_S + Sf_I \right) - \sigma f_E \tag{17}$$

Substitute:

$$\frac{df_E}{dt} = \frac{\beta}{N}\left[I\left(-\frac{\beta SI}{N} \right) + S\left(\sigma E - \gamma I \right) \right] - \sigma\left(\frac{\beta SI}{N} - \sigma E \right) = -\frac{\beta^2 SI^2}{N^2} + \frac{\beta S}{N}\left(\sigma E - \gamma I \right) - \frac{\sigma\beta SI}{N} + \sigma^2 E \tag{18}$$

For I :

$$\begin{aligned} \frac{df_I}{dt} &= \sigma\frac{dE}{dt} - \gamma\frac{dI}{dt} = \sigma f_E - \gamma f_I \\ &= \sigma\left(\frac{\beta SI}{N} - \sigma E \right) - \gamma\left(\sigma E - \gamma I \right) \\ &= \frac{\sigma\beta SI}{N} - \sigma^2 E - \gamma\sigma E + \gamma^2 I \\ &= \frac{\sigma\beta SI}{N} - E\left(\sigma^2 + \gamma\sigma \right) + \gamma^2 I \end{aligned} \tag{19}$$

For R :

$$\frac{df_R}{dt} = \gamma\frac{dI}{dt} = \gamma f_I = \gamma\left(\sigma E - \gamma I \right) \tag{20}$$

Thus, the numerical solution of Influenza cases is :

$$\begin{aligned} S_{n+1} &= S_n + h\left(-\frac{\beta SI}{N} \right) + \frac{h^2}{2}\left[\frac{\beta^2 SI^2}{N^2} - \frac{\beta S}{N}\left(\sigma E - \gamma I \right) \right] \\ E_{n+1} &= E_n + h\left(\frac{\beta SI}{N} - \sigma E \right) + \frac{h^2}{2}\left[-\frac{\beta^2 SI^2}{N^2} + \frac{\beta S}{N}\left(\sigma E - \gamma I \right) - \frac{\sigma\beta SI}{N} + \sigma^2 E \right] \\ I_{n+1} &= I_n + h\left(\sigma E - \gamma I \right) + \frac{h^2}{2}\left[\frac{\sigma\beta SI}{N} - E\left(\sigma^2 + \gamma\sigma \right) + \gamma^2 I \right] \\ R_{n+1} &= R_n + h\left(\gamma I \right) + \frac{h^2}{2}\left[\gamma\left(\sigma E - \gamma I \right) \right] \end{aligned} \tag{21}$$

3. Results and Discussion

This chapter discusses the results of the numerical simulation of the SEIR model for influenza cases. The Second-Order Taylor Series method and the Fourth-Order Runge-Kutta method are applied to solve the model. To evaluate the accuracy and solution behaviour of the two methods, the numerical results are compared and analysed. The simulation results of the basic SEIR model using the Second-Order Taylor Series method and the Fourth-Order Runge-Kutta method are displayed in this section. Over the span of the simulation, changes in Infectious (I) populations will be shown and analysed. The influenza case data used in this study were obtained from published research. Monthly influenza case data for 2018 were taken from [3], which analysed influenza cases in Malaysia using official surveillance records. The data represent confirmed infected cases and were used

to set the initial condition of the infected population and to analyse infection trends in the SEIR model. No primary data collection was conducted, as this study focuses on numerical method comparison.

By categorizing the population into susceptible, exposed, infected and recovered groups, the basic SEIR model used in this study gives a simplified framework to describe the dynamics of influenza transmission. Considering the scope of this research, the model is appropriate to illustrate the general trend of disease transmission and to provide a numerical comparison between the Second-Order Taylor Series method and RK4 method. Yet the model has several limitations. It doesn't take consideration of demographic factors such as birth, natural death, migration, vaccination or seasonal effects; instead, it assumes homogenous population mixing and constant parameter values. Additionally, there may be differences between the predicted and actual infection trends due to model's depends on assumed parameter values and the limited reported case data. Therefore, rather than representing an accurate prediction of real-world outbreaks, the result should be considered as an approximation of influenza dynamics.

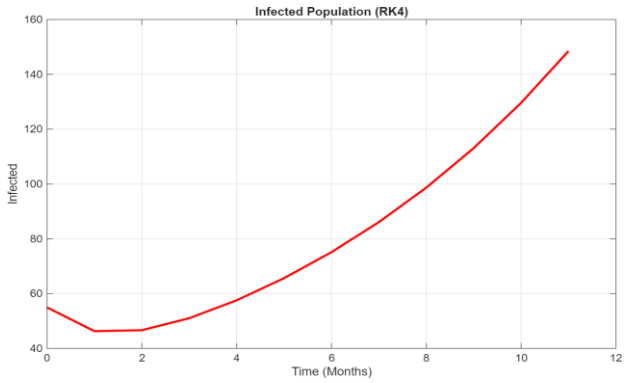

Fig. 2 Prediction of Infected (I) using RK4 Method

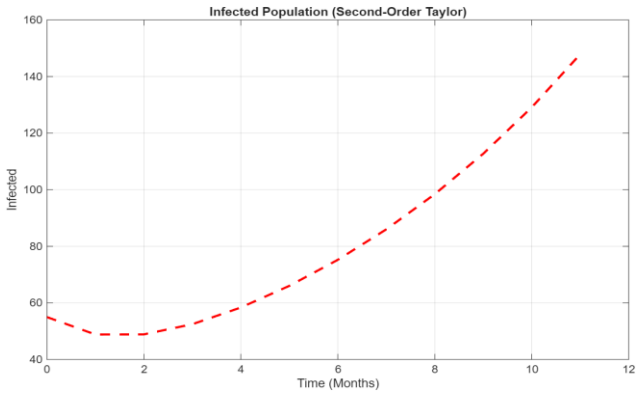

Fig. 3 Prediction of Infected (I) using Second-Order Taylor Series Method

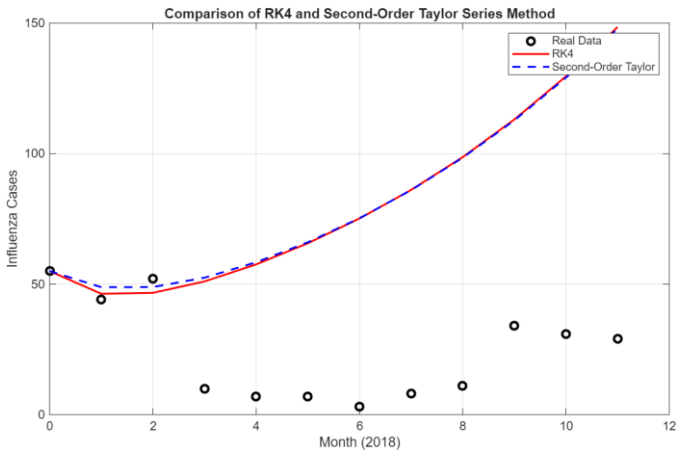

Fig. 4 Comparison of RK4 and Second-Order Taylor Series Method

Table 3 Monthly Predicted Infected Cases (I)

Month	Predicted Infected Cases using RK4 Method	Predicted Infected Cases using Second-Order Taylor Series Method
January	55.00	55.00
February	46.32	48.84
March	46.67	48.88
Aprill	51.04	52.48
May	57.57	58.38

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June	65.65	66.03
July	75.14	75.24
August	86.09	86.00
September	98.67	98.44
October	113.08	112.73
November	129.59	129.12
December	148.48	147.88

In Fig. 2 and Fig. 3, starting from an initial infected value of 55, this graph shows the predicted number of infected individuals over time (months) using the RK4 method and Second-Order Taylor Series method. Like the Fourth Order Runge-Kutta method, the number of infected cases slightly declined at the start of the simulation before increasing gradually over the next few months. This shows that both methods are capturing the same general disease dynamics and solving the same SEIR. This trend demonstrates that both methods can capture the general dynamics of disease transmission and represent the typical progression of an influenza outbreak.

However, there is noticeable difference between the two methods, particularly during the peak infection period. In comparison to the Second-Order Taylor Series method, which shows small deviations, the RK4 method produces a smoother curve and more stable peak. These differences happen because the infected compartment is more sensitive to numerical approximation errors due to its rapid changes during the outbreak. The RK4 method gives greater accuracy and stability for handling these sudden changes by assessing the model at multiple intermediate points within each time step. On the other hand, the Second-Order Taylor Series method uses lower-order approximations, that result in slightly greater errors. The error analysis, which indicates smaller Mean Absolute Error than the Second-Order Taylor Series method, indicating higher precision in forecasting the infected population, further supports this behaviour.

In Fig. 4, the real influenza case data and the numerical results from both methods are shown in this comparison graph. Early in the simulation, the RK4 and Second-Order Taylor Series curves are very near to each other, showing that both predictions perform effectively when predicting short-term behaviours. There's a little difference between the two curves as time moves on, and RK4's trajectory maintains smoother. Each numerical curve is fully followed by the real data points. This difference is expected since real-world elements such as seasonal effects, public health interventions, or the behavioural changes are not included in the basic SEIR model. Therefore, the comparison favours numerical stability and accuracy over perfect matching with real data.

Considering susceptible, exposed, and recovered populations cannot be directly observed, real influenza surveillance data only limited to confirmed cases, so error analysis in this study is mainly focused on the infected population. The expected outcomes are not precisely consistent with the actual data because the basic SEIR model is a simplified representation of actual disease transmission and ignores factors like seasonality, behavioural changes, public health interventions, or demographic effects. Because both numerical methods are applied to the same model, parameters, and initial conditions, meaning that any differences in the study are due to numerical methods themselves rather than the data or model assumptions, the comparison continues to be valid despite these differences.

Compared to the RK4 method, the Second-Order Taylor Series method requires fewer calculations at each time step, which makes it computationally simpler. The RK4 method costs longer to compute because it involves multiple intermediate evaluations. However, when the number of infected individuals changes rapidly, RK4's higher cost of processing allows it to produce more accurate and stable results. Despite the computation time difference is small for the simulation scale used in this study, RK4 is more accurate and reliable overall.

4. Conclusion

Based on the results and discussion, this study compared the performance of the Fourth-Order Runge-Kutta method and the Second-Order Taylor Series method in solving the basic SEIR model for influenza cases. The comparison remains valid even though the model predictions did not exactly correspond to real influenza data because both methods were applied under similar modelling conditions. Based on the results, the RK4 method generated a lower Mean Absolute Error of 57.7458 compared to 57.8553 for the Second-Order Taylor Series method, showing that RK4 method provides slightly better numerical reliability and accuracy. As a result, it turns out that Fourth-Order Runge-Kutta method is more appropriate for solving the SEIR model of influenza transmission in this study.

The exact solution of the SEIR model is obtained by solving the system of ordinary differential equations analytically with given initial conditions. This solution represents the continuous-time behaviour of the epidemic model without numerical approximation. The exact solution is used as a reference to assess the

accuracy of the Euler and Runge–Kutta methods. A similar approach to obtaining exact solutions for SEIR-type models is discussed in [11].

There are several limitations to this study. Age structure, seasonality, vaccination, and public health interventions are some of the factors that could impact real influenza transmission dynamics and aren't taken into consideration by the basic SEIR model, that assumes homogeneous mixing of the population. Furthermore, because the model parameters are obtained from existing literature rather being properly calibrated to Malaysia's influenza data, differences between the simulated results and actual reported cases could happen. Furthermore, this study employs monthly influenza case data, while the SEIR model operates in continuous time, which might lead to inconsistencies between the observed data and the model predictions

There are multiple various ways to make this study better. To more accurately represent real influenza transmission and improve precision in forecasting, future studies could utilize more detailed epidemiological model involving demographic factors, seasonal effects, or vaccination. To improve accuracy, stability, and computational efficiency, different numerical techniques such as higher order Taylor Series methods, implicit methods, or multistep methods can be studied and compared with the RK4 method. Finally, to reduce uncertainty and increase the reliability of model predictions, parameter calibration using more detailed or higher-resolution data, such as weekly or daily influenza case data, is recommended.

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Conflict of Interest

Authors declare that there is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Mohammad Zhulqarnaim Hamizan, Mahathir Mohamad;* ***data collection:*** *Mohammad Zhulqarnaim Hamizan;* ***analysis and interpretation of results:*** *Mohammad Zhulqarnaim Hamizan, Mahathir Mohamad;* ***draft manuscript preparation:*** *Mohammad Zhulqarnaim Hamizan, Mahathir Mohamad. All authors reviewed the results and approved the final version of the manuscript*

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