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A Constructive Heuristic Continuous Approach for Solving Uncapacitated Location-Allocation Problem With Zone-Dependent Fixed Cost

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Abstract

Location analysis is a fundamental area within Operations Research that focuses on identifying optimal facility locations and allocating customer demand in a manner that balances operational efficiency and cost minimization. This study explores the uncapacitated continuous location-allocation problem with zone-dependent fixed costs, where variations in regional fixed costs introduce additional complexity into the decision-making process. Owing to the NP-hard nature of the multisource Weber problem, obtaining exact solutions becomes computationally intractable, particularly for large-scale problem instances. To overcome these challenges, this research applied a constructive heuristic approach as the Multi-Start Version of the Modified Cooper's Method (MS-MCM). The algorithm was implemented using Python programming, and its performance was evaluated in comparison with the standard Modified Cooper's Method and Simulated Annealing (SA), a well-established metaheuristic renowned for its capability to effectively explore large solution spaces. The computational results demonstrate that MS-MCM outperforms the benchmark methods, achieving the lowest total cost of 215.52, compared to 218.28 obtained by the standard Modified Cooper's Method and 196.73 achieved by Simulated Annealing. These findings indicate that MS-MCM is both an effective and practical solution approach for addressing uncapacitated continuous location-allocation problems with zone-dependent fixed costs.

1. Introduction

Location analysis, commonly referred to as facility location analysis, is a well-established area of research in the literature [1]. However, the concept of sustainable location analysis has received comparatively limited attention. This study aims to define this concept more clearly and to propose a structured framework for addressing sustainable location decision-making. Location analysis constitutes a critical field within Operations Research and logistics, focusing on three fundamental questions: where to locate facilities, how many facilities to establish, and how to allocate customer demand to these facilities. Traditionally, location analysis problems are formulated to minimize total cost, which represents a central goal of classical facility location models [2]. These decisions are influenced by various factors, including distance, cost, demand, and operational constraints.

The primary goal of this study is to simultaneously minimize total cost, comprising transportation and fixed costs, and enhance service performance, such as reducing delivery time, improving accessibility, and increasing customer coverage. Location analysis models are generally classified into three categories: continuous or discrete models, and capacitated or uncapacitated models [3]. This study focuses on the Uncapacitated Continuous Location–Allocation Problem (UCLAP) with zone-dependent fixed costs. Fixed costs represent expenses that are incurred regardless of the volume of output or demand served, while zone-dependent fixed costs indicate that these costs vary across different geographic zones. In the context of UCLAP with zone-dependent fixed costs, facilities may be located anywhere within a continuous space, and no capacity limitations are imposed on the facilities.

The Location–Allocation Problem (LAP) involves determining the locations of one or more facilities, with the multi-facility case being considerably more complex both conceptually and mathematically [4]. Key considerations in LAP include transportation costs and fixed costs, where fixed costs are influenced by factors such as land prices, construction expenses, taxation, zoning regulations, infrastructure availability, and haulage costs. These factors can significantly affect the final location decisions. The constructive heuristic approach is commonly employed to approximate solutions for NP-hard problems, where obtaining exact optimal solutions for large-scale instances is computationally infeasible. Such heuristics construct solutions incrementally until a complete solution is formed [5].

UCLAP is also known as the Multisource Weber Problem (MSWP), which involves locating multiple facilities in a continuous space to minimize the total cost of serving a set of demand points. Large-scale instances of this problem cannot be efficiently solved using exact methods, as the presence of multiple facilities and zone-dependent fixed costs further increases problem complexity. Consequently, heuristic and metaheuristic methods are often adopted, as they provide practical and efficient solution approaches for real-world problems under computational constraints [5]. Nevertheless, many existing methods either do not fully account for zone-dependent fixed costs or are prone to convergence at local optima, highlighting the need for a more robust and efficient heuristic capable of handling continuous spaces, multiple facilities, and heterogeneous fixed costs.

To address these challenges, this study establishes two primary objectives. First, it aims to solve the uncapacitated continuous location–allocation problem with zone-dependent fixed costs using a Multi-Start Version of the Modified Cooper's Method (MS-MCM). Second, it evaluates the performance of MS-MCM by comparing it with two benchmark methods: Simulated Annealing [6] and the standard Modified Cooper's Method [7]. The Modified Cooper's Method is further refined by incorporating Algorithm 1 [8], which enhances solution quality at each iteration when dealing with zone-dependent fixed costs. The proposed MS-MCM improves solution robustness for continuous location–allocation problems, particularly non-convex and NP-hard cases, by repeatedly initiating the search from different starting points until convergence is observed, defined as obtaining identical solutions across three consecutive runs.

The computational experiments are conducted using the widely recognized 50-customer benchmark dataset introduced by [9], which provides a standardized basis for performance evaluation and validation of the proposed algorithm. The objective is to determine the optimal locations of two facilities. The heuristic framework of the Multi-Start Modified Cooper's Method, supported by Algorithm 1, is implemented, and the solution procedures are coded using the Python programming language.

2. Research Method

This part is an overview of constructive heuristics. It starts with a basic problem of finding the best location for a single facility such as a warehouse or factory. The setup cost varies from zone to zone. After setting the basic problem, proceeding with the more complex problem of locating the multiple facilities at the same time.

The location of facility is influenced by two types, which are costs and transportation cost. The objective of this analysis is to identify a balance between two aspects. The mathematical models employed in location analysis are typically divided into three categories: network models (where the solution space, X consists of a finite union of linear, continuous sets), discrete models (suitable for finite solution spaces, X), and continuous models (where $X \subseteq \Re^q$)[10].

Table 1 defines the core mathematical symbols used to formulate the location-allocation problem and it provides the essential foundation for understanding the subsequent mathematical model and the objective of minimizing the total of these costs.

Table 1 Nomenclature

Symbol	Description
x	Location of the facility
m	Number of Facility
n	Number of customer or demand points

$f(x)$	Location fixed cost
$c(x,y)$	Transportation cost from location n to demand y

2.1 Problem Formulation

The Uncapacitated Continuous Location-Allocation Problem (UCLAP) with zone-dependent fixed costs involve locating M facilities in a continuous plane to serve n customers, such that the total cost for comprising both of transportation and zone-dependent fixed costs to minimize. The mathematical model is given as follows [6]:

Given:
Parameters:

n : the number of fixed points (or customer points);
w_j : demand or weight of customer $j(j=1,\dots,n)$;
$a_j : (a_j^1, a_j^2)$: location of customer j where $a_j \in \Re^2, (j=1,\dots,n)$;
M : the number of fixed points (or customer points).

Decision Variables:

x_f, y_f : coordinates of the facility, where $x_f, y_f \in \Re$.

Derived Variables:

q_j : quantity assigned from the facility to the customer j .

Objective:
Minimize the total cost given by:

$$\text{Total Cost} = \sum_{j=1}^{M}\sum_{j=1}^{n} x_{ij} d(x_i, a_j) + \sum_{i=1}^{M} f(x_i) \tag{1}$$

where:
$c(a_j, x_f, y_f)$ represents the distance between the facility f and customer j .

Subject to

$$\sum_{i=1}^{M} x_{ij} = w_j, \forall_j = 1,\dots,n \tag{2}$$

$$x_{ij} \geq 0, \forall_i = 1,\dots,M; j = 1,\dots,n \tag{3}$$

(2) ensure that each customer's demand is met.
(3) prevents negative quantities or distances.

The setup makes no restriction whatsoever on the amount shipment that can be handled at all, which means the customer are assigned to the nearest facilities, with arbitrarily broken ties. However, with the opening of each facility, a fixed cost is incurred, depending on the location.

2.2 Distance Metric

From location-allocation problems, the accurate measurement of distances is critical. There have two common distance metrics are used on this [11]. Rectilinear Distance is use in grid-like urban environments, where the travel is constrained by the streets (known as a Manhattan or city-block distance) while Euclidean Distance is a straight-line distance, commonly used in open, less structured areas. This study primarily focuses on the

Manhattan distance because of its practical relevance in grid-based environments but however, the Euclidean distance is also considered in certain cases, such as in the application of Weiszfeld's equation as proposed in [12].

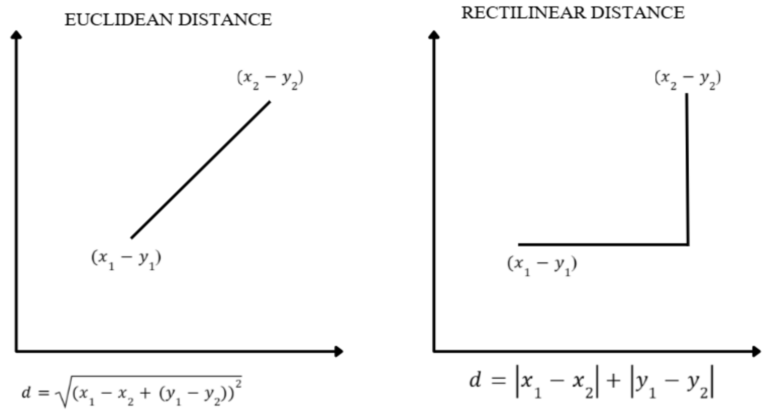

Fig. 1 *Rectilinear and Euclidean Distance* [12]

This study focuses on Manhattan distance because of its practical application in grid-based environments, but the Euclidean distance will be considered like in the application of Weiszfeld's equation in a way proposed by [13].

2.3 Solution Approach for Single-Facility Case

To understand the multi-facility problem, we need to solve the simpler single-facility problem with zone-dependent fixed costs.

2.3.1 Weiszfeld Equation

This equation was proposed by Endre Vaszonyi Weiszfeld in 1936 and it is a well-known method to find the best location of a single facility minimizing the total weighted Euclidean distance between the facility and existing infrastructure. That equation is iterative, improving the location estimate through successive computations. To find the optimal location of a single facility, we need to apply this Weiszfeld equation iteratively:

$$x_f(k+1) = \frac{\sum_{j=1}^{n} d_j . x_j}{\sum_{j=1}^{n} d_j} \text{ and } y_f(k+1) = \frac{\sum_{j=1}^{n} d_j . y_j}{\sum_{j=1}^{n} d_j} \tag{4}$$

where k represents the iteration number. The process continues until the location converges.

2.3.2 Algorithm 1

Algorithm 1 [13] is developed for finding the optimal location for a single facility in the non-existence of zone dependent fixed costs. The goal is to minimize both fixed costs and transport:

$$\text{Minimize } \sum_{j=1}^{n} c(a_j, x_f, y_f).q_j + \sum_{f=1}^{m} f(x_f, y_f) \tag{5}$$

The algorithm proceeds with the following steps:

Step 1: Solve the single-facility minisum problem and check if the solution lies in the zone with the Smallest fixed cost.

Step 2: For each candidate polygon, determine its visibility and boundary.

Step 3: Use a one-dimensional interval bisection search to find the optimal facility location.

This method involves solving the location problem iteratively while adjusting for zone-dependent fixed costs.

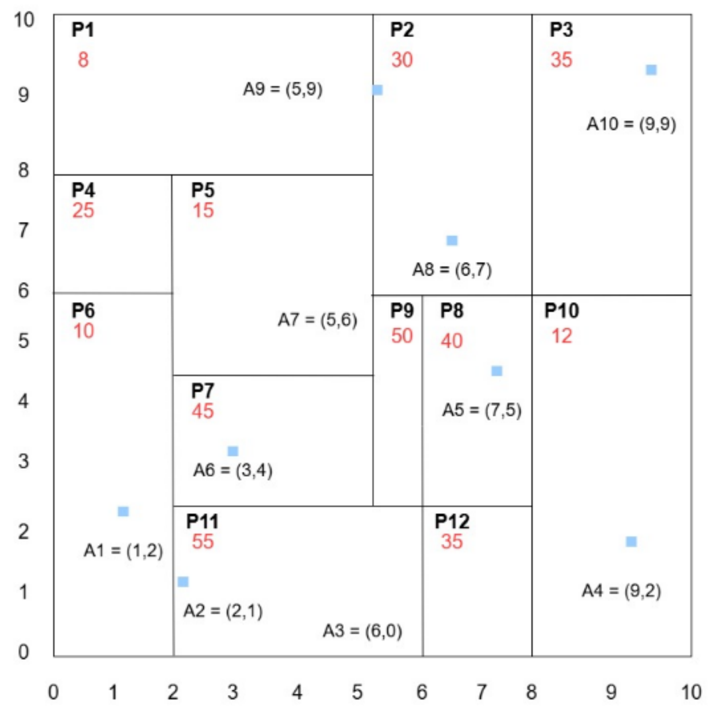

Fig. 2 *Numerical Example of Algorithm 1* [13]

Fig. 2 is a numerical examplw that has illustrated for Algorithm 1. The square is divided into several rectangular zones, with specific customer locations. The algorithm begins with solving the single facility minisum problem using the Weiszfeld equation and subsequently selects the optimum facility location accordingly to the zone fixed costs.

2.4 Multi-Facility Location-Allocation Problem with Zone-Dependent Fixed Cost with Multi-Start Version of Modified Cooper's Method

This method is known as repetition of Modified Cooper's Method. The Multi-Start Version of Modified Cooper's Method (MS-MCM) is the type of constructive heuristic that will be used for solving the multi-facility problem to get the better result from the standard Modified Cooper's Method's result. The procedure is constructed to obtain the partial solution gradually until a complete solution is produced while keeping an eye on the objective value. Below are the steps of MS-MCM as follows:

Step 1: Choose m initial facility locations randomly.

Step 2: Allocate for each customer to its nearest facility.

Step 3: Reallocate the facilities by solving the m single-facility location problem by using Algorithm 1.

Step 4: Repeat **Step 2** to **Step 3** until no further improvement is achieved over three successive iterations.

Step 5: Repeat **Step 1** to **Step 4** for K times (until a limit on execution time is reached); retain the best solution from all local minimum thus obtained.

3. Results and Discussion

This study uses the 50-point dataset from [9] as test case for focusing on a scenario with two facilities (m=2). This represents the implementation process of the Multi-Start Version of Modified Cooper's Method, discuss the results obtained, and provides insights into both of strengths and weaknesses of three methods in solving the Uncapacitated Continuous Location-Allocation Problem with Zone-Dependent Fixed Costs efficiently.

3.1 Computational Result

3.1.1 Best Run (Run 5) – Multi-Start Version of Modified Cooper's Method

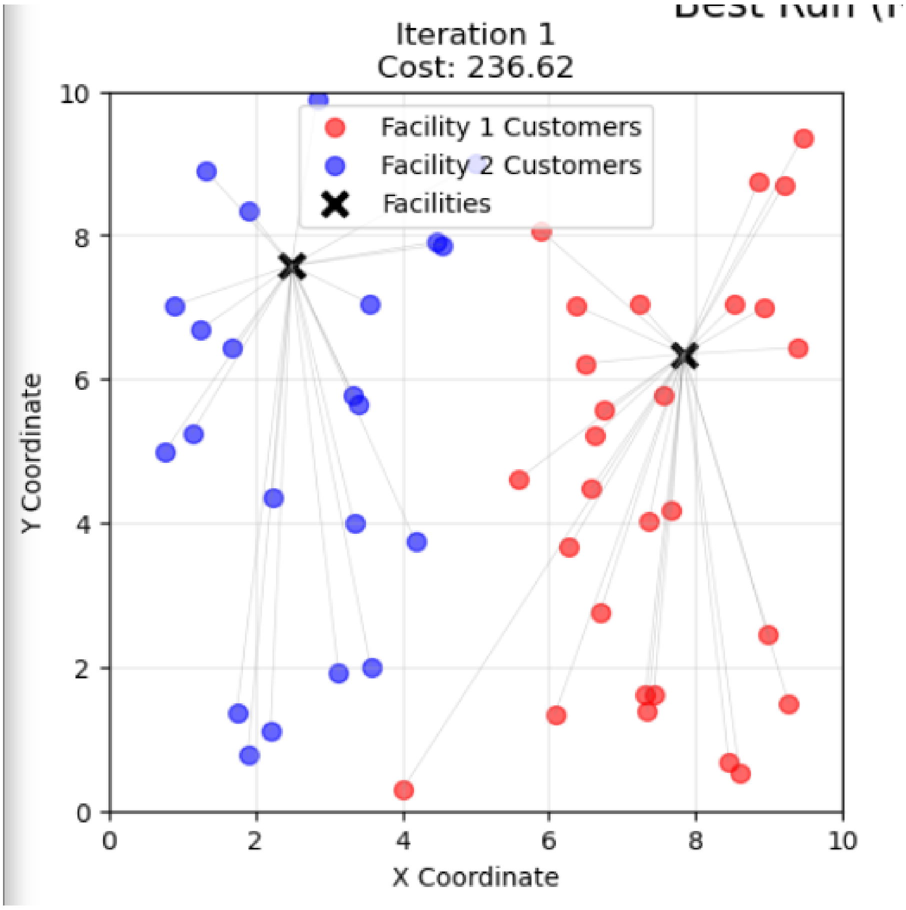

Fig. 3 Iteration 1

Fig. 3 to Fig. 5 display a detailed description of the iterative optimization process. The blue and red points represent customer locations at facility 1 and facility 2. The black points represent facility locations at each iteration, which indicates the process by which the facilities adjust their positions throughout optimization. Additionally, the grey lines connect the customer locations with their assigned facilities showing visually where customers are assigned facilities for each iteration. These connections show the reallocation of the customers to facilities as the facility locations change to lower total costs.

Table 2 Multi-Start Version of Modified Cooper's Method Iteration (Best Run = 5)

Iteration	Total Cost
1	215.52
2	215.52
3	215.52

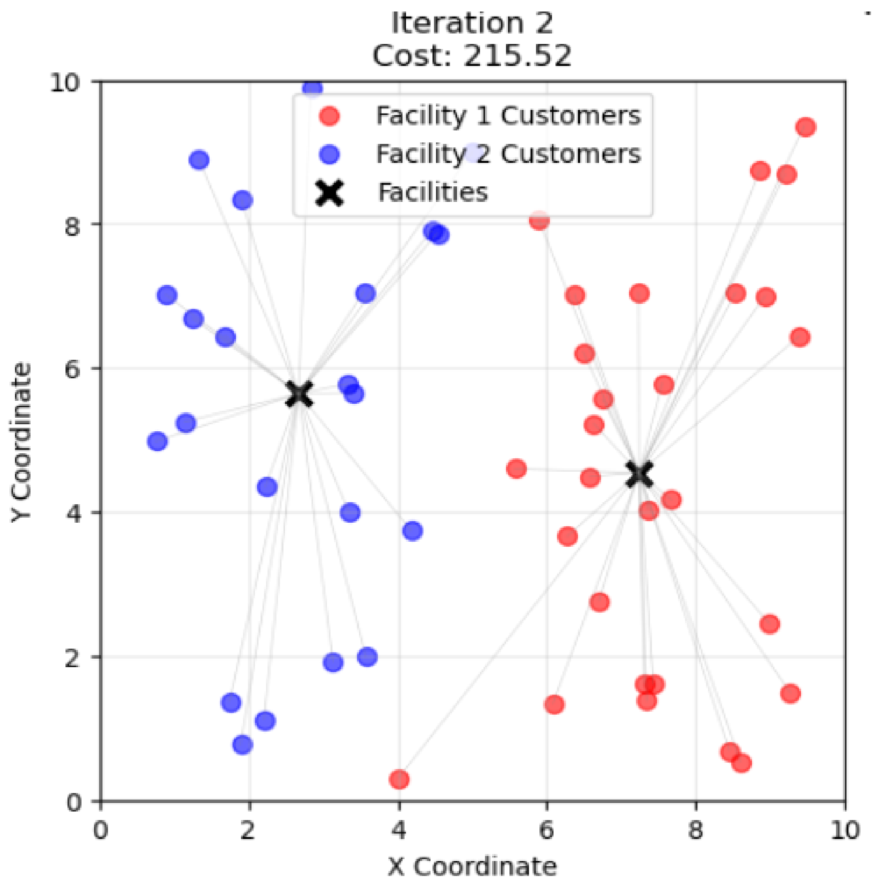

Fig. 4 Iteration 2

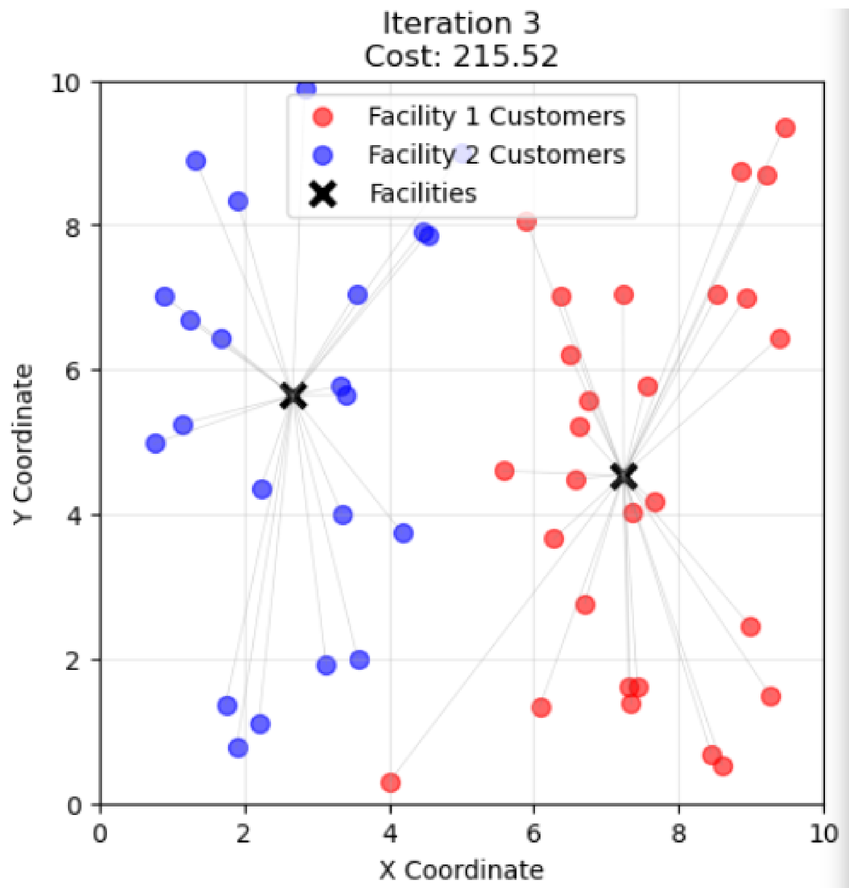

Fig. 5 Iteration 3

Table 2 shows the Multi-Start Version of Modified Cooper's Method iteration at K = 5. The algorithm getting stuck in poor local optimum. Starting from facilities at (7.84, 6.35) and (2.49, 7.58), the total cost converged immediately after the first iteration with a high cost of 215.52. The "early convergence" occurred because the facility locations didn't change between iterations, indicating the algorithm found a local minimum where customer allocations didn't improve when facilities were reallocated. Despite being worse than previous runs, this became the new "global best" because the previous best was 215.54, showing how multi-start explores different solutions but can sometimes settle on suboptimal points. This demonstrates the importance of multiple runs to escape poor local minima.

Table 3 Summary of all runs

Run	Total Cost	Iterations
1	215.59	8
2	220.29	4
3	220.29	7
4	215.55	7
5	215.52	3

For Table 3, MS-MCM produced varying results across 5 runs, with costs ranging from 215.52 to 220.29, demonstrating significant sensitivity to initial random placements. The best solution emerged in Run 5 with the lowest cost of 215.52, converging quickly in just iterations, while other runs required up to 8 iterations to settle into their respective local optima. This variability highlights that facility location problems contain multiple local minima and Run 5 fortuitously found the most favourable starting configuration. The results validate the MS-MCM approach value, as a single random initialization might have yielded a significantly worse solution like the 220.29 from Run 2 or 3.

3.2 Comparison of MS-MCM with standard Modified Cooper's Method and Simulated Annealing (SA)

Table 4 Comparison of each method

Method	Facility location	Cost
Multi-Start Version of Modified Cooper's Method	(2.67,5.64) and (7.24,4.54)	215.52
Standard Modified Cooper's Method	(2.79,6.89) and (6.66,3.95)	218.28
Simulated Annealing	(2.00,5.65) and (8.00,4.60)	196.73

The comparison between the Multi-Start Version of Modified Cooper's Method with standard Modified Cooper's Method and Simulated Annealing approach, as applied by [6], will reveals the distinct trade-offs in associated costs and facility placement.

Table 4 compares the performance of three solutions approaches like Multi-Start Version of Modified Cooper's Method (MS-MCM), standard Modified Cooper's Method (MCM), and Simulated Annealing (SA) in term of facility locations obtained and the corresponding objective function values of 196.73, indicating the best overall solution quality for the problem instance considered.

The superior performance Simulated Annealing can be attributed to its stochastic search mechanism, which allows occasional acceptance of worse solutions with a certain probability during the search process. This probabilistic acceptance enables the algorithm to effectively escape local optima and explore a broader region of the solution space, thereby increasing the likelihood of identifying high-quality or near-global optimal solutions. As a result, SA demonstrates stronger global search capabilities compared to deterministic local search methods.

The MS-MCM yields a cost of 215.52, outperforming the Standard Modified Cooper's Method, which records a slightly higher cost of 218.28. This improvement highlights the benefit of incorporating multiple random initial solutions in the MS-MCM. By restarting the algorithm from different starting points, MS-MCM reduces sensitivity to initial conditions and mitigates the risk of convergence to poor local minima. However, despite this enhancement, the method remains constrained by the deterministic nature of the underlying Modified Cooper's Method, which limits its ability to escape locally optimal solutions once convergence occurs.

Overall, the results illustrate a fundamental trade-off between solution quality and algorithmic approach. Stochastic metaheuristics such as Simulated Annealing tend to produce better-quality solutions due to their global search behaviour, albeit at the expense of higher computational complexity and parameter tuning requirements. In contrast, deterministic methods like the Modified Cooper's Method and its Multi-Start variant offer simpler implementation and faster convergence but may yield inferior solutions in complex, non-convex problem landscapes.

4. Conclusion

This study, we implemented and compared three facility location optimization methods: the Standard Modified Cooper's Method, its Multi-Start version, and Simulated Annealing, using a dataset of 50 customer locations with zone-dependent fixed costs. The Standard Modified Cooper's Method demonstrated the fundamental local search approach, efficiently converging to local optima but showing sensitivity to initial conditions with costs around 218.28. The Multi-Start Version of Modified Cooper's Method improved upon this by running multiple random initializations, achieving a better cost of 215.52 by exploring different regions of the solution space and selecting the best result across runs. However, both deterministic variants remained constrained by their tendency to converge with local minimum without mechanisms to escape them.

Significantly, Simulated Annealing outperformed both Cooper's Method variants with a substantially lower cost of 196.73, highlighting the advantage of stochastic global optimization techniques for facility location problems. This superior performance stems from Simulated Annealing's ability to accept temporarily worse solutions during search, enabling it to escape local optima and explore the solution space more comprehensively. These findings underscore an important trade-off in selecting an optimization strategy: while deterministic methods like Cooper's Method offer simplicity and faster convergence, stochastic methods like Simulated Annealing provide better solution quality at the expense of a more complex implementation and potentially longer runtime, making them preferable when solution quality is paramount in complex facility location scenarios.

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Conflict of Interest

Authors declare that there is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Nuralya Natashah Zaabar;* ***solve equation:*** *Nuralya Natashah Zaabar;* ***analysis and interpretation of results:*** *Nuralya Natashah Zaabar; Azila Md Sudin;* ***draft manuscript preparation:*** *Nuralya Natashah Zaabar; Azila Md Sudin.*

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