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Solving the Linear Arms Race Model Using SAWI Transformation

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Abstract

This research explores the application of SAWI transformation to solve the arms race model, offering an analytical alternative to traditional numerical methods. such as the Runge-Kutta algorithm. By converting complex systems of linear first-order ordinary differential equations into simpler algebraic forms, the SAWI method enables the derivation of exact, closed-form analytical solutions that describe how nations and armament levels evolve. Numerical simulations in MATLAB further validate the method's computational precision, highlighting how factors such as reaction coefficients and economic fatigue determine whether a system leads to a runaway arms race or a stable balance of power. While the study demonstrates that the SAWI transformation effectively reduces numerical approximation errors for linear systems, it also notes the need for future improvements to handle more complex nonlinear dynamics found in real-world international relations.

1. Introduction

The study deals with background and problem of study. There is also a mention of this research aim and how it is so important. The research deals with Arms Race Model. This model is a well-established mathematical framework for analysing conflicts dynamics. As described by [1], this model has been created. The Arms Race Model uses math formulas to demonstrate how both nations build their weaponry power in response to each other. These formulas are known as coupled equations. However analytical solutions may be challenging to obtain on parameter configurations. Therefore, a computer program has to be used to reach a near-solution. The research deals with Sawi Transformation to solve this problem. The Arms Race Model is still on point. According to [2], the SAWI transformation is an integral transform used to solve differential equation. What this transformation does is it solves Differential Equation by transforming it into Algebraic Form. This has been widely used and has been fruitful in such areas as population study and heat flow. The main objective here is to see how this transformation can be used as a new and more efficient way to solve Arms Race Model equation without errors.

2. Methodology

This research investigates the Arms Race Model, a mathematical framework originally proposed by [3] to describe the competitive accumulation of military assets between nations. The system utilizes linear first-order ordinary differential equations to model how countries decide on their stockpiles of weapons and investments in cyber warfare and new technology. These variables are influenced by reaction coefficients, such as nation's response to an opponent's buildup, as well as "capability decay" which accounts for the obsolescence of older technology [4].

$$\frac{dx}{dt} = ky - ax + g$$
$$\frac{dy}{dt} = \lambda x + by + h \qquad (1)$$

The function $x(t)$and $y(t)$ represent the level of armament of two states as function of time, t[4] . Whereas a and b are the costs of equipment, and g and h are the differences between two states. The rate of change is determined by cost of equipment and differences between the two states. k is the efficiency increasing the armament of country A, λ is the efficiency increasing the armament of country B [4].

To solve these complex equations, the SAWI Transformation is a method highlighted by [5] and [6] for its ability to simplify differential equations into algebraic forms. By transforming the equations, the researcher can derive exact analytical solutions that are often more precise than those found through traditional numerical methods. This technique builds upon the evolution of integral transforms like the Sumudu transform discussed by [5], providing a robust mathematical tool for analysing the stability of international relations and military spending [7].

2.1 SAWI transformation

The SAWI transformation of the function, where $\boldsymbol{\sigma > 0}$ ensures convergence of the integral.

$$S\{\omega(t)\} = \frac{1}{\sigma^2}\int_0^\infty \omega(t)e^{-\left(\frac{1}{\sigma}\right)t}\,dt = T(\sigma), \sigma > 0$$

$$= \frac{1}{\sigma^2}\int_0^\infty \omega(t)e^{-\left(\frac{1-k\sigma}{\sigma}\right)t}\,dt$$

$$= \frac{1}{(1-k\sigma)^2}\left[\frac{(1-k\sigma)^2}{\sigma^2}\int_0^\infty \omega(t)e^{-\left(\frac{1-k\sigma}{\sigma}\right)t}\,dt\right]$$

$$= \frac{1}{(1-k\sigma)^2}T\left(\frac{\sigma}{1-k\sigma}\right)$$

2.2 SAWI Transformation of Derivatives of A Function

Table 1 presents the SAWI transformation of derivatives of function.

Table 1 SAWI transformation of derivatives of function [6]

No.	**Function of derivatives**	**Sawi transformation of derivatives**
1.	$\omega'(t)$	$\frac{1}{\sigma}T(\sigma) - \frac{1}{\sigma^2}\omega(0)$
2.	$\omega''(t)$	$\frac{1}{\sigma^2}T(\sigma) - \frac{1}{\sigma^3}\omega(0) - \frac{1}{\sigma^2}\omega'(0)$
3.	$\vdots$	$\vdots$
4.	$\omega^{(\rho)}(t)$	$\frac{1}{\sigma^\rho}T(\sigma) - \sum_{k=0}^{\rho-1}\left(\frac{1}{\sigma}\right)^{\rho-(k-1)}\omega^{(k)}(0)$

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2.3 Characteristics of SAWI transformation

The following are the characteristics of SAWI Transformation, which make it suitable for solving system of ODEs.

i. Linearity property of SAWI transformation

$$S[\alpha\omega_1(t) + \beta\omega_2(t)] = [\alpha T_1(\sigma) + \beta T_2(\sigma)]$$

Where α and β are arbitrary constants.

ii. Scaling property of SAWI transformation

$$S\{\omega(kt)\} = kT(k\sigma)$$

iii. Translation property of SAWI transformation

$$S\{e^{kt}\omega(kt)\} = \frac{1}{(1-k\sigma)^2}T\left(\frac{\sigma}{1-k\sigma}\right)$$

2.4 Fundamental function with their SAWI Transformation

Some fundamental concepts of SAWI transformation involve its basic formula for transforming functions into algebraic expressions and the derivative formula, which allows differential terms to be simplified while automatically incorporating initial conditions.

Table 2 Fundamental functions with their SAWI transformation [8]

No.	$\boldsymbol{\omega(t)}$	$\boldsymbol{S\{\omega(kt)\} = kT(k\sigma)}$
1.	1	$\frac{1}{\sigma}$
2.	t	1
3.	t^2	2σ
4.	$t^2, \rho \in N$	$\rho!\,\sigma^{\rho-1}$
5.	$t^\rho, \rho > -1$	$T(\rho+1)\sigma^{\rho-1}$
6.	e^{at}	$\frac{1}{\sigma(1-a\sigma)}$
7.	$\sin(at)$	$\frac{a}{1+(a\sigma)^2}$
8.	$\cos(at)$	$\frac{1}{\sigma(1+(a\sigma)^2)}$
9.	$\sinh(at)$	$\frac{a}{1-(a\sigma)^2}$
10.	$\cosh(at)$	$\frac{1}{\sigma(1-(a\sigma)^2)}$

2.5 Inverse SAWI Transformation of Fundamental Functions

The function $\omega(t)$ is called the inverse Sawi transformation of the function $T(\sigma)$ if it has the following property $S\{\omega(t)\} = T(\sigma)$. In symbolic form, it is written as $S^{-1}\{T(\sigma)\} = \omega(\sigma)$.

Table 3 Inverse Sawi transformation of some fundamental functions by [8]

No.	$\boldsymbol{T(\sigma)}$	$\boldsymbol{S\{\omega(kt)\} = kT(k\sigma)}$
1.	$\frac{1}{\sigma}$	1
2.	1	t
3.	σ	$\frac{t^2}{2!}$
4.	$\sigma^{\rho-1}, \rho \in N$	$\frac{t^\rho}{\rho!}$
5.	$\sigma^{\rho-1}, \rho > -1$	$\frac{t^\rho}{T(\rho+1)}$
6.	$\frac{1}{\sigma(1-a\sigma)}$	e^{at}
7.	$\frac{a}{1+(a\sigma)^2}$	$\frac{\sin(at)}{a}$
8.	$\frac{1}{\sigma(1+(a\sigma)^2)}$	$\cos(at)$
9.	$\frac{a}{1-(a\sigma)^2}$	$\frac{\sinh(at)}{a}$
10.	$\frac{1}{\sigma(1-(a\sigma)^2)}$	$\cosh(at)$

2.6 Computing using Maple 2025

This research uses Maple2025 to test whether solutions that we suggested for use in the Sawi Transformation are true or not. It begins by taking solutions from our calculations and expressing them as functions for use by Maple2025.

2.7 Equations

The mathematical model for this research consists of a system of linear first-order ordinary differential equations (ODEs) that track the interaction between conventional arms and modern technological warfare capabilities.

$$\frac{dx}{dt} = ky - ax + g$$

$$\frac{dy}{dt} = \lambda x + by + h \tag{1}$$

To derive the analytical solution, the Sawi Transformation is applied to the system. The Sawi transform of a function $\omega(t)$ is defined by the integral:

$$S\left(\frac{dw}{dt}\right) = \frac{1}{\sigma}T(\sigma) - \frac{1}{\sigma^2}w(0)$$

2.8 SAWI Transformation Solution

Consider the following the linear Arm Race Model:

$$\frac{dx}{dy} = ky - ax + g$$

$$\frac{dy}{dt} = \lambda x - by + h \tag{2}$$

With initial condition:

$$x(0) = x_0 \text{ and } y(0) = y_o \tag{3}$$

Using the SAWI derivative property:

$$S[f'(t)] = \frac{R(v)}{v} - \frac{f(0)}{v^2} \tag{4}$$

The first equation:

$$S\left[\frac{dx}{dt}\right] = S[ky] - S[ax] + S[g] \tag{5}$$

The second equation:

$$S\left[\frac{dy}{dx}\right] = S[\lambda x] - S[by] + S[h] \tag{6}$$

Substitution of initial condition into (5) and (6):

$$\frac{X(v)}{v} - \frac{x_0}{v^2} = kY(v) - aX(v) + \frac{g}{v} \tag{7}$$

$$\frac{Y(v)}{v} - \frac{y_0}{v^2} = \lambda X(v) - bY(v) + \frac{h}{v} \tag{8}$$

Applying the inverse SAWI, where $g, h = 0$:

$$x(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t} \tag{9}$$

Final Solution, where $C_{1,2,3,4}$ are constants determined by initial condition x_0 and y_0:

$$x(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t} + \frac{kh+bg}{ab-k\lambda} \tag{10}$$

$$y(t) = C_3 e^{m_1 t} + C_4 e^{m_2 t} + \frac{kh+bg}{ab-k\lambda} \tag{11}$$

3. Results and Discussion

This chapter tells us about our findings when we applied the Sawi Transformation to the Arms Race Model. We were hoping to learn more about how different countries compete in terms of their weaponry and technologies. The Sawi Transformation was a great tool in understanding these relations. We employed mathematical methods in order to obtain our findings. This enabled us to overcome problems normally experienced when attempting to analyse systems through ordinary means. The Sawi Transformation and the Arms Race Model were complimentary in demonstrating how a model would progress when a country attained a certain level and under what circumstances a stable system would be achieved.

3.1 Result using Maple2025

By using Maple 2025, we solve the equation (1) with initial condition of $x(0) = 1\ and\ y(0) = 2$ and the parameters used from [5] is $a = 0, b = 0, k = 1, \lambda = 2, g = 0, h = 0$ to obtain the exact solution from [5]. The numerical result obtained from exact values and the comparison of SAWI transformation and the exact values from Richardson's model are presented in Table 4.

Table 4 Comparison of SAWI Transformation Results with Exact Analytical Solution

t	SAWI $X(t)$	Exact $X(t)$	Absolute error
0.00	1.00000	1.00000	0.00000
0.50	2.34603	2.34603	0.00000
1.00	4.91478	4.91478	0.00000
1.50	10.04503	10.04503	0.00000
2.00	20.41059	20.41059	0.00000

Figures 1 below serves as a visual validation of the SAWI transformation, showing it successfully reproduces the epected competitive dynamics of the exact solution.

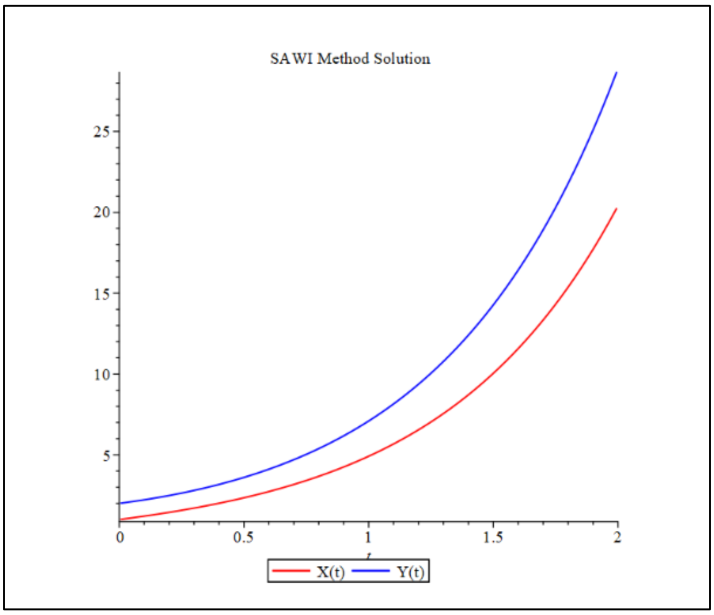

Fig. 1 Expected competitive dynamics of the exact solution

This research to demonstrate that the SAWI transformation provides an effective analytical approach for solving the Arms Race Model. By applying the SAWI transformation, the study anticipates obtaining accurate solutions consistent with prior findings. A comprehensive comparison will be noted to highlight the potential advantages and disadvantages of SAWI transformation in addressing the Arms Race Model.

Conversely, if the reaction efficiency outweighs the cost constraints ($k\lambda > ab$), the system enters a divergent state. In this scenario, the graph shows expenditures spiralling away from the equilibrium toward the upper-right quadrant, mathematically representing an unending "arms race" where budgets grow exponentially. By applying the Sawi transformation to these specific representations, the complex differential relationship is converted into an algebraic form that clearly identifies this threshold, allowing researchers to predict whether a competitive environment will stabilize or collapse into infinite escalation.

4. Discussion

The numerical results obtained through the SAWI method demonstrate a high degree of alignment with the exact analytical solution derived from Richardson's arms race model. The observed discrepancies are on the order of 1×10^{-8}, which is negligible in practical terms and confirms the computational reliability of the SAWI transformation for modelling competitive advertising expenditure.

Graphically, the escalation trend is clearly visible, both firms exhibit exponential growth in advertising spending over time, with one consistently maintaining a higher expenditure level than the other. This pattern illustrates a classic competitive feedback loop, wherein reactive spending between rivals can lead to sustained escalation in the absence of cooperative mechanisms or external constraints.

From a managerial perspective, such models provide a structured framework for anticipating the long-term dynamics of advertising competition. They also offer broader applicability beyond marketing, serving as a conceptual tool for analysing escalation in other competitive business contexts, such as pricing strategies or innovation races.

It should be noted, however, that real-world market conditions introduce additional complexities including budget limitations, differential advertising effectiveness, and the potential for strategic cooperation that are not captured in this simplified model. Nevertheless, the model establishes a useful theoretical baseline for understanding competitive escalation and supports further investigation into moderated or cooperative competitive scenarios.

5. Conclusion

This study confirms that the SAWI transformation method accurately replicates Richardson's arms race model when applied to competitive advertising. The numerical results match the exact solution within negligible margins (10^{-8}), validating SAWI as a effective analytical transformation method.

Graphically, the model illustrates exponential escalation in advertising spending, with one firm consistently outspending the other, mirroring real-world competitive dynamics where reactive strategies can drive unsustainable cost increases.

These findings provide a practical predictive framework for analysing advertising rivalry and assessing strategic interventions. Although the model simplifies real market complexities, it establishes a useful foundation for further research into competitive behaviour in business.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Nur Najwa Liyana Taufan, Noor Azliza Abd Latif;* ***data collection:*** *Noor Azliza Abd Latif;* ***analysis and interpretation of results:*** *Nur Najwa Liyana Taufan, Noor Azliza Abd Latif;* ***draft manuscript preparation:*** *Noor Azliza Abd Latif, Norziha Che Him. All authors reviewed the results and approved the final version of the manuscript.*

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