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Solving Two Species Lotka-Volterra Biological Model Using Sawi Transformation

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Abstract

The paper explores the use of Sawi Transformation to solve the two species Lotka-Volterra predator-prey model, which is ruled by an ordinary differential equation (ODE) system that is nonlinear. The main aim is to show how this fundamental transformation is working to simplify nonlinear systems through the reduction of the complexity of the system by transforming it into algebraic equations to enable semi-analytical solutions with less computation effort. Maple 2015 is used to obtain iterative solutions to both the classical and modified Lotka-Volterra models. The findings are then compared the Variational Iteration Method (VIM). The comparison proves that the Sawi Transformation method has highly accurate results which are equally efficient. These results confirm that Sawi Transformation is a robust valid instrument in solving nonlinear dynamical systems with a lot of success in ecological and biological modelling.

1. Introduction

The Lotka-Volterra model, also known as the predator-prey model is a classic mathematical model in ecological models, which models the interaction between two biological species. This model was introduced separately by Alfred Lotka and Vito Volterra in 1926, and is a system of nonlinear first-order ordinary differential equations that are used to model the population dynamics of predators and their prey [1]. Although the Lotka-Volterra model is a relatively simple nonlinear system, it is difficult to find the precise analytical solution of the system which often involves applying some approximate or numerical methods [2]. There are several other analytic and numeric methods to solve the Lotka-Volterra equations, such as Variational Iteration Method (VIM), Runge-Kutta-Fehlberg (RKF) method, and Laplace Adomian Decomposition Method (LADM). The main aim of this paper is to use the Sawi Transformation in getting semi-analytic solutions of the classical and modified Lotka-Volterra models. It is also assessed in terms of the accuracy and reliability in terms of the Variational Iteration Method, which has been extensively used as a standard technique to solve nonlinear biological models [2], [3].

Integral transformation methods have gained more and more popularity in recent years in solving ordinary differential equations because of their potentials of simplifying complex systems. Various researchers have demonstrated that different equations can be transformed into algebraic equations in an experiential way using integral transforms, which has less complexity in terms of computation [4]. One of such methods is a more recent integral transform, the Sawi Transformation, which is an integral of Fourier-like types, and which has been shown to have strong potential to solve linear and nonlinear ordinary differential equations [4], [5]. Graphical analyses and numerical simulations are performed to prove the usefulness of the Sawi Transformation in the process of capturing the key predator-prey dynamics. The Sawi Transformation has convenient features especially in the working with initial conditions and derivative functions, which reduce it to initial value problems [5].

2. Methodology

The Sawi transform is used to solve two systems regarding the application of Sawi transform method in the classic and modified Lotka-Volterra model

2.1 Lotka-Volterra Models

2.1.1 Classic Lotka-Volterra Model

The classic Lotka-Volterra model is presented as in Equations (1) and (2) [1]:

$$\frac{dx}{dt} = x(t)\big(a - by(t)\big), \tag{1}$$

$$\frac{dy}{dt} = y(t)(dx(t) - c), \tag{2}$$

where x is the prey population, y is the predator population, a, b, c, d are the positive real constant and t is time

2.1.2 Modified Lotka-Volterra Model

The modified Lotka-Volterra model outlined from the study [2]:

$$\frac{dN_1}{dt} = N_1(b_1 + a_{11}N_1 + a_{12}N_2), \tag{3}$$

$$\frac{dN_2}{dt} = N_2(b_2 + a_{21}N_1 + a_{22}N_2), \tag{4}$$

where, N_1 is the prey population, N_2 is the predator population, $a_{11}, a_{12}, a_{21}, a_{21}, b_1, b_2$ is the positive real constants, with t as the time. The competition between two species that inhabit the same ecological niche is described by equations (3) and (4).

2.2 Sawi Transformation

2.2.1 Definition and Basic Properties

The Sawi transform of a function $\boldsymbol{\omega(t)}$, defined for $\boldsymbol{t} \geq \boldsymbol{0}$, is given by Equation (5):

$$\boldsymbol{S\{\omega(t)\} = \frac{1}{\sigma^2}\int_0^\infty \omega(t)\, e^{-t/\sigma} dt = T(\sigma), \sigma > 0,} \tag{5}$$

where $\boldsymbol{\sigma}$ is the transform variable. This definition ensures the transform exists for functions of exponential order. Key properties include:

2.2.2 Sawi Transformation of derivatives

$$\boldsymbol{\omega'(t) = \frac{1}{\sigma}T(\sigma) - \frac{1}{\sigma^2}\omega(0).} \tag{6}$$

2.3 Application to Lotka-Volterra Systems

2.3.1 Transform of the Classic Model

Using the derivative property of the Sawi transformation given in Equation (6), the Sawi transforms of $\frac{dx}{dt}$and $\frac{dy}{dt}$are expressed as follows:

$$\boldsymbol{S\left\{\frac{dy}{dt}\right\} = \frac{1}{\sigma}X(\sigma) - \frac{1}{\sigma^2}x(0),} \tag{7}$$

$$S\left\{\frac{dy}{dt}\right\} = \frac{1}{\sigma}Y(\sigma) - \frac{1}{\sigma^2}y(0), \tag{8}$$

where $X(\sigma) = S\{x(t)\}$ and $Y(\sigma) = S\{y(t)\}$ denote the Sawi transforms of the prey and predator population functions respectively.

Applying the Sawi transform Equations (7) and (8) to the classical Lotka–Volterra system in Equations (1) and (2) yields:

$$\frac{1}{\sigma}X(\sigma) - \frac{1}{\sigma^2}x_0 = \alpha X(\sigma) - \beta S\{xy\}, \tag{9}$$

$$\frac{1}{\sigma}Y(\sigma) - \frac{1}{\sigma^2}y_0 = \delta S\{xy\} - \gamma Y(\sigma). \tag{10}$$

2.3.2 Application to Modified Model

Using the derivative property of the Sawi transformation, the time-derivative terms are transformed as follows:

$$S\left\{\frac{dN_1}{dt}\right\} = \frac{1}{\sigma}T_1(\sigma) - \frac{1}{\sigma^2}N_1(0), \tag{11}$$

$$S\left\{\frac{dN_2}{dt}\right\} = \frac{1}{\sigma}T_2(\sigma) - \frac{1}{\sigma^2}N_2(0), \tag{12}$$

where:

$T_1(\sigma) = S\{N_1(t)\}$ and $T_2(\sigma) = S\{N_2(t)\}$ $\boldsymbol{which\ is\ the\ Sawi\ transform\ of\ N_1(t)\ and\ N_2(t), respectively}$. $N_1(t)$ and $N_2(t)$ are the population of species 1 and 2 at time t.

The Sawi transformation of Equation (15) and (16) are applied to the modified Lotka–Volterra system given in Equations (3) and (4) and becomes:

$$\frac{1}{\sigma}T_1(\sigma) - \frac{1}{\sigma^2}N_{10} = b_1T_1(\sigma) + a_{11}S\{N_1^2\} + a_{12}S\{N_1N_2\}, \tag{13}$$

$$\frac{1}{\sigma}T_2(\sigma) - \frac{1}{\sigma^2}N_{20} = b_2T_2(\sigma) + a_{21}S\{N_1N_2\} + a_{22}S\{N_2^2\},$$

where N_{10} *and* N_{20} are the initial populations of species 1 and 2. b_1 and b_2 are the intrinsic growth rates of species 1 and 2. a_{11} and a_{22} are the intraspecies competition coefficients (self-interaction). a_{12} is the effect coefficient of species 2 on species 1. a_{21} is the effect coefficient of species 1 on species 2. (14)

2.4 Implementation and Computational Framework

All calculations were done using Maple 2015 software, chosen for its symbolic computation abilities and solid handling of special functions. The process involved defining the Sawi transform and inverse transform procedures symbolically, setting up automated iteration loops with checks for convergence, numerically assessing solutions at specific time points, graphing population trajectories and phase portraits, and analysing errors by comparing with reference solutions. Parameter values and initial conditions were taken from established literature [1] and [2] to allow direct comparison with existing methods.

3. Results and Discussion

3.1 Example of Case Studies and Parameter Sets

3.1.1 Example 1: Classic Model with Specific Parameters (Case 1)

Following [1], parameters were set as $\alpha = 0.1, \beta = 1, \delta = 1, \gamma = 1$, with initial conditions $x(0) = 14, y(0) = 18$. These values represent a system with relatively slow prey growth and strong interaction coefficients, leading to pronounced oscillations. Figure 1 presents the numerical values at discrete time points, comparing Sawi transform results with VIM solutions from literature. Numerical Comparison for Case 1 (t = 0 to 0.1 with $\boldsymbol{h} = \mathbf{0.01}$)

Table 1 Numerical solution of Sawi transform and VIM [1] in Case 1

t	**Sawi, *X***	**VIM, *X***	**Absolute Error, *X***	**Sawi, *Y***	**VIM, *Y***	**Absolute Error, *Y***
0.00	14.00000000	14.00000000	0.000e+00	18.00000000	18.00000000	0.000e+00
0.01	11.57385194	11.57385194	0.000e+00	20.24746002	20.24746002	0.000e+00
0.02	9.384867540	9.384867520	1.900e-08	22.23344016	22.23344016	0.000e+00
0.03	7.549236440	7.549236380	6.300e-08	23.84334054	23.84334054	0.000e+00
0.04	6.183148310	6.183148160	1.490e-07	24.96256128	24.96256128	0.000e+00
0.05	5.402792790	5.402792500	2.910e-07	25.47650250	25.47650250	0.000e+00
0.06	5.324359540	5.324359040	5.030e-07	25.27056432	25.27056432	0.000e+00
0.07	6.064038220	6.064037420	7.990e-07	24.23014686	24.23014686	0.000e+00
0.08	7.738018470	7.738017280	1.193e-06	22.24065024	22.24065024	0.000e+00
0.09	10.46248996	10.46248826	1.700e-06	19.18747458	19.18747458	0.000e+00
0.10	14.35364233	14.35364000	2.330e-06	14.95602000	14.95602000	0.000e+00

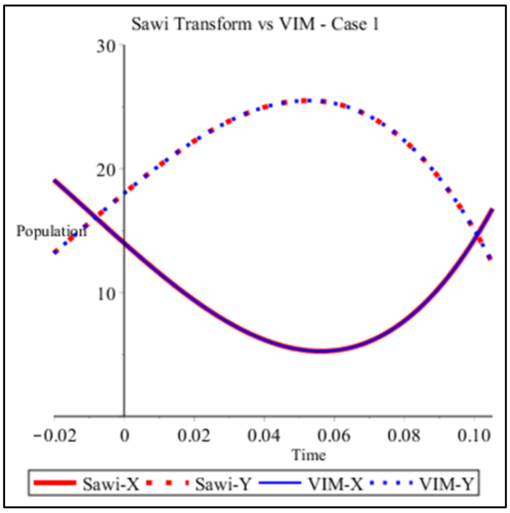

Fig. 1 The comparison of the results by Sawi transforms and VIM [1] in Case 1

From Fig. 1 and Table 1, it is evident that the results show minimal discrepancy. This indicates that the application of the Sawi transforms yields outcomes nearly identical to those obtained with the Variational Iteration Method (VIM) when solving the Lotka-Volterra equations. Both methods employed a four-term approximate solution. The observed trend where an increase in Y corresponds to a decrease in X reflects the expected ecological dynamics that is a rise in the predator population leads to a decline in the prey population.

3.1.2 Example 2: Classic Model with Specific Parameters (Case 2)

A second parameter set with $\alpha = 1$, $\beta = 1$, $\delta = 1$, $\gamma = 0.1$ and initial conditions $x(0) = 16$, $y(0) = 10$ was analyzed to test method robustness. Figure 2 presents the numerical values at discrete time points, comparing Sawi transform results with VIM solutions from literature. Numerical Comparison for Case 2 ($t\ =\ 0$ to 0.1 with $h\ =\ 0.01$)

Table 2 Numerical solution of Sawi transforms and VIM [1] in Case 2

t	Sawi, X	VIM, X	Absolute Error, X	Sawi, Y	VIM, Y	Absolute Error, Y
0.00	16.00000000	16.00000000	0.000e+00	10.00000000	10.00000000	0.000e+00
0.01	14.50420240	14.50420241	1.000e-08	11.63757646	11.63757648	2.000e-08
0.02	12.92321920	12.92321925	5.000e-08	13.34299172	13.34299182	1.000e-07
0.03	11.29666480	11.29666496	1.600e-07	15.07527456	15.07527488	3.200e-07
0.04	9.664153600	9.664153980	3.840e-07	16.79345376	16.79345453	7.700e-07
0.05	8.065300000	8.065300750	7.500e-07	18.45655812	18.45655962	1.500e-06
0.06	6.539718400	6.539719700	1.296e-06	20.02361644	20.02361903	2.590e-06
0.07	5.127023200	5.127025260	2.058e-06	21.45365750	21.45366161	4.110e-06
0.08	3.866828800	3.866831870	3.072e-06	22.70571008	22.70571622	6.140e-06
0.09	2.798749600	2.798753970	4.374e-06	23.73880298	23.73881173	8.750e-06
0.10	1.962400000	1.962406000	6.000e-06	24.51196500	24.51197700	1.200e-05

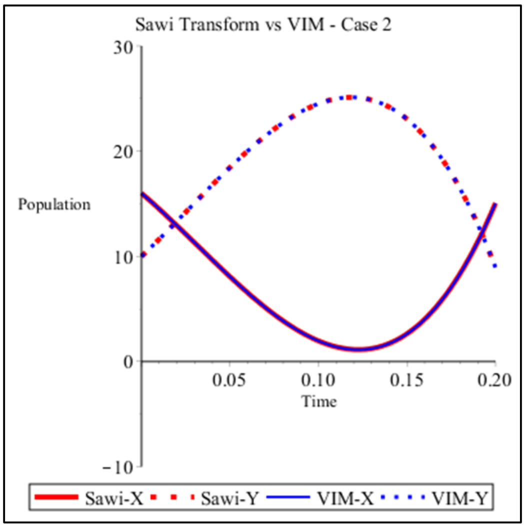

Fig. 2 The trend of the results by Sawi transforms and VIM [1] in Case 2

Upon examination of Fig. 2 and Table 2, it is clear that the results show very small differences. This indicates that the Sawi transform and the Variational Iteration Method (VIM) produce highly similar outcomes when applied to the Lotka-Volterra model. Both methods employed a four-term approximate solution in their calculations. As illustrated in Fig. 2, an increase in the predator population Y corresponds with a decrease in the prey population X. This observed relationship reflects the fundamental predator-prey dynamic in which a rise in predators leads to a decline in prey numbers.

3.1.3 Example 3: Modified Lotka-Volterra model

For the modified model with parameters from [2]:$a_{11} = -0.0014$,$a_{12} = -0.0012$,$a_{21} = -0.0009$,$a_{22} = -0.001$, $b_1 = 0.1$,$b_2 = 0.08$, and initial conditions $N_1(0) = 4$, $N_2(0) = 10$. Remarkably, from the result obtained, the VIM solutions exactly matched to machine precision (10^{-10}), validating the method for more complex model structures. Numerical Comparison for this example ($t = 0$ to 1.0 with $h = 0.1$)

Table 3 Numerical solutions of Sawi transforms and 2-iteration VIM [2]

t	Sawi, N_1	2-iteration VIM, N_1	Absolute Error, N_1	Sawi, N_2	2-iteration VIM, N_2	Absolute Error, N_2
0.0	4.000000000	4.000000000	0.000e+00	10.000000000	10.000000000	0.000e+00
0.1	4.033070747	4.033070747	0.000e+00	10.066572500	10.066572500	0.000e+00
0.2	4.066363461	4.066363461	0.000e+00	10.133490290	10.133490290	0.000e+00
0.3	4.099878843	4.099878843	0.000e+00	10.200753850	10.200753850	0.000e+00
0.4	4.133617599	4.133617599	0.000e+00	10.268363670	10.268363670	0.000e+00
0.5	4.167580434	4.167580434	0.000e+00	10.336320190	10.336320190	0.000e+00
0.6	4.201768050	4.201768050	0.000e+00	10.404623900	10.404623900	0.000e+00
0.7	4.236181154	4.236181154	0.000e+00	10.473275250	10.473275250	0.000e+00
0.8	4.270820449	4.270820449	0.000e+00	10.542274730	10.542274730	0.000e+00
0.9	4.305686638	4.305686638	0.000e+00	10.611622800	10.611622800	0.000e+00
1.0	4.340780429	4.340780429	0.000e+00	10.681319930	10.681319930	0.000e+00

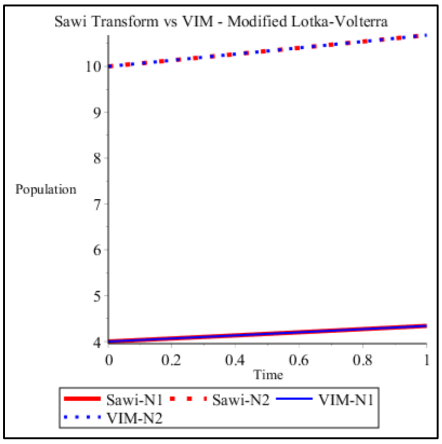

Fig. 3 The comparison of Sawi transform and 2-iteration VIM [2]

As seen in Fig. 3 and Table 3, the rise in N_1 is accompanied by an increase in N_2, illustrating that prey population growth is followed by predator population growth, consistent with predator–prey dynamics. For the modified model, population trajectories show logistic-type growth with stabilizing tendencies. Both populations approach steady-state values rather than oscillating indefinitely, reflecting the stabilizing influence of intraspecific competition terms $(a_{11}, a_{22} < 0)$. The Sawi transform successfully captures this asymptotic behaviour, with solutions converging to equilibrium. Complementing this, Figure 1 demonstrates that an increase in Y corresponds to a decrease in X, underscoring the inverse relationship in which a larger predator population results in a decline in prey numbers.

4. Conclusions and Recommendations

Finally, the aims of the present research have been met. The Sawi transformation has been successfully applied to the classical and modified version in resolving the two species Lotka-Volterra model. A methodology of systematic iterative process was used to obtain semi-analytical solutions of the polynomials, which had a clear understanding of predator-prey dynamics, such as oscillatory behaviour and equilibrium behaviour. The nonlinear system can be simplified to a more manageable algebraic form by the transformation to allow accurate analytical or semi-analytical solutions. Also, numerical comparisons with the Variational Iteration Method (VIM) prove the Sawi transformation gives solutions of the same quality and reliability results. The good result match indicates that the Sawi-based method is successful in the process of capturing nonlinear predator-prey population interactions.

Although these are encouraging outcomes, there are a number of constraints. The Sawi transform is still relatively new, and its implementation has so far been restricted to particular classes of nonlinear ordinary differential equations. Future research is suggested to improve the approach, combining it to other mathematical

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approaches, like hybrid formulations using either Laplace transform or Adomian polynomial methods, to better deal with nonlinear terms. Also, the Sawi transformation can be applied to more sophisticated ecological systems, such as multispecies systems, time- delay effects or spatial dynamics, which would also prove the strength of Sawi transformation. Its role as a useful tool in computational and ecological mathematics would also be enhanced by comparative studies of a wider variety of established numerical and analytical methods.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Nur Zawani Jamaludin:* ***Analysed and interpreted data.*** *Nur Zawani Jamaludin, Noor Azliza Abd Latif, Norziha Che Him. All authors reviewed the results and approved the final version of the manuscript.*

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