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Multivariate Analysis (MA) of Train Swaying Dynamics Based on Wheel Parameters and Operational Speed

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Abstract

The aim of this research is to isolate the effect of two basic mechanical degradation mechanisms in the wheels, namely the thickness of the flanges and the diameter of the wheels, on the overall train swaying behavior, as well as to quantify the influence of the kinematic operational speed of the train. The study was based on historical maintenance data sets as well as on dynamic field measurements of 28 different observations. Multivariate analysis, with Multiple Linear Regression (MLR) model, was used to process data. The model was validated to ensure reliability and to normalize any prediction errors with Mean Squared Error (MSE) which yielded a very low error of 0.0017. The statistical analysis gave a coefficient of determination of R^2 = 0.201 and the individual P-values for flange thickness (0.456), wheel diameter (0.142) and operational speed (0.087) were all higher than 0.05. The results clearly demonstrate that wheel wear and the current operating speeds are not the main contributor towards train swaying in this operational envelope. A mathematical validation of the strong intrinsic stability of the bogie system is obtained for this finding. Finally, future mitigation strategies will need to move beyond wheel reprofiling and focus on the higher-order dynamics that require investigation.

1. Introduction

The railway transport system is increasingly focussing on high operational speeds, with large demands on train dynamic behaviour and system safety. Train swaying is an ongoing challenge in train dynamics. When the carbody sways excessively in the lateral, yaw or roll directions, the comfort for the passengers is directly affected and, in extreme cases, the operational safety can be severely affected. This dynamic instability is not caused by one individual factor but the interaction of the wheel profile, track geometry and operating speed. The wheel-rail contact geometry is completely changed by natural wheel wear and certain levels of equivalent conicity can cause high-frequency bogie hunting or low-frequency carbody swaying. At the same time, the vibration excitations to the vehicle are continuous as the wear and the static track parameters are altered. In addition, the operational speed is the key parameter for these instabilities. Swaying is generally more pronounced at higher speeds when the track irregularity frequency is directly in the rigid-body mode of the vehicle. Since train swaying is a multi-parameter problem, the maintenance measures applied so far, which focus only on the wheel profile, for example,

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wheel reprofiling, can be of varying effectiveness. Hence, a statistical, multivariate method must be used to identify and quantify the combined effect of these basic parameters on the overall stability of the train. Although the interaction between trains and track is a complex phenomenon, existing maintenance practices have not adequately considered lateral instability as a countermeasure and have focused on the wheel reprofiling process as scheduled maintenance. Such a unique approach, however, often produces erratic mitigation outcomes, highlighting an important missing link in the knowledge of what truly drives swaying kinematically. This research has set out to mathematically eliminate and examine the actual effect of these parameters for this operational deficit. To address this operational deficiency, this research aims to mathematically isolate and evaluate the true influence of these parameters. Specifically, the objectives of this study are threefold: first, 1. To identify the parameters that contribute towards train swaying behavior; second, to analyze the parameters using Multivariate Analysis to assess the train swaying behavior.; and third, to formulate a multivariate linear regression model to quantify the combined interaction of wheel condition parameters and operational speed on the overall train swaying behavior.

2. Literature Review

Train swaying, or lateral hunting motion, is a crucial dynamic phenomenon which is governed by the very highly nonlinear mechanics of the wheel-rail contact interface [1, 2]. Natural wear changes the equivalent conicity of the contact patch while the wheels are in operation, and consequently, the mode of instability of the vehicle. Hollow wear is seldom found in medium or severe forms and is normally accompanied by unnormally high equivalent conicity. This can cause high frequency bogie hunting, which results in increased wear of wheels and rails, increased lateral forces, and drastically shortened wheel life [3] [4]. On the other hand, low equivalent conicity is a major factor leading to low frequency carbody hunting. This type of instability is very disturbing and can easily be amplified by the carbody suspension modes and drastically reduce passenger ride comfort [5].

Wheel reprofiling is a routine maintenance practice that is carried out to restore dynamic performance and correct wheel-rail contact in vehicles. The main method of restoring a standard profile, including the LMA profile, is to reduce the lateral forces on the wheel-rail that are caused by the flange, and consequently the eccentricity in the wheel [6]. When the flange thickness is reduced or they are being used on newly reprofiled wheels, however, the resulting equivalent conicity [6] or [7] is often abnormally low. This unintentionally makes the carbody extremely vulnerable to low frequencies of the vehicle system in the initial stages of the reprofiling life cycle, as the low frequencies are close to the lowest damping ratio of the carbody [5].

In addition, these instability modes are the "kinematic catalysts" of the operational speed. The dynamic response of the train is very speed dependent: at higher transit velocities, the vehicle tends to get closer to the critical hunting velocity, so the vibrations become more nonlinear due to the wheel-rail interface [8]. The effects of individual wheel profile wear and equivalent conicity under simulated conditions are well documented in existing literature [9] but the train swaying under active field operating conditions is the combined effect of physical degradation of the wheel (flange thickness and wheel diameter) and varying operating speeds. This is underscoring the need to mathematically separate these parameters and mathematically consider their combined kinematic effect on the lateral train swaying using a multivariate statistical approach.

3. Research Flowchart

The dynamic field measurements were systematically gathered through scheduled monthly observation sessions across Express and Transit trainsets operating on Tracks 2 and 3. During these physical data collection procedures, train swaying was recorded utilizing an MTi-G-710 sensor from Xsens, securely positioned at the center of the driver's cabin door entrance and monitored via Xsens MT Manager 2019.2 software. Measurements were logged dynamically, with targeted readings isolated at specific kilometer markers (including KM 21.7, 26.5, 33, 36, 37.7, 40.7, 49, and 52.5) identified by overhead catenary line signboards. Concurrently, kinematic speed data was actively acquired in real-time from the train's Driver Display Unit (DDU). Following the field sessions, technicians calculated average swaying values by extracting the highest and lowest points from the live measurement graphs. These average values were subsequently merged with historical wheel profile degradation records to formulate the comprehensive numerical dataset required for the Microsoft Excel statistical evaluation.

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Fig.2 MTi-G-710 sensor from Xsens

The research methodology begins with the simultaneous collection of three primary field datasets: Wheel Data, Speed Data, and Swaying Data. Once collected, these datasets undergo a Data Sorting phase where they are merged, cleaned, and organized into a structured format suitable for mathematical evaluation. The organized data is then fed into Microsoft Excel for Data Analysis, which acts as a critical validation checkpoint. If there are formatting errors or the data is not ready for evaluation, the process triggers a "NO" loop, sending it back to the Data Sorting phase for correction. However, if the statistical analysis is successfully executed ("YES"), the process advances to Results Interpretation, where the generated statistics are evaluated to form engineering conclusions. Finally, these findings are compiled during the Results Writing phase to form the final thesis document, marking the completion of the methodological process.

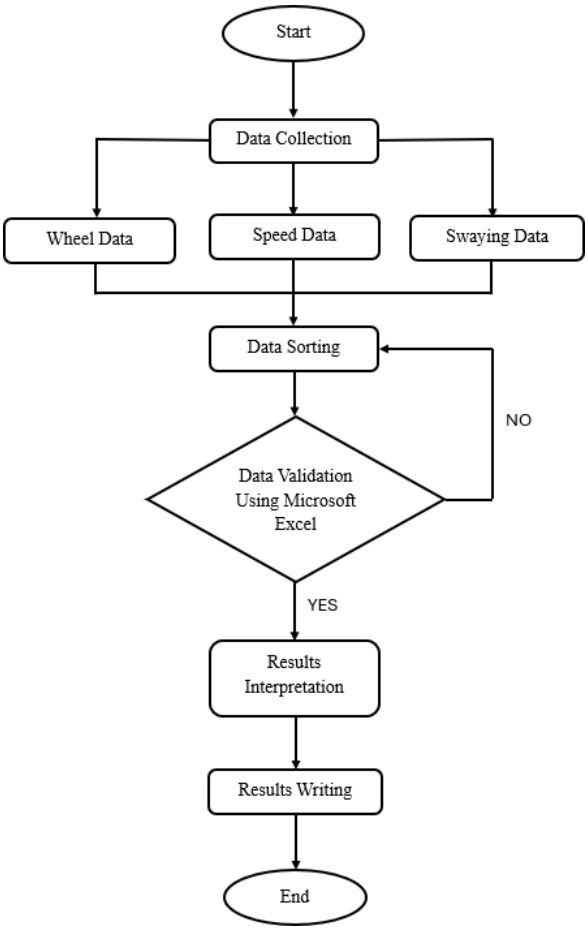


Fig.2: Flow Chart of Research Methodology

3.1 Data Analysis

The result of this extraction and sorting has been attached in the appendix section where the finalized and consolidated data set is presented as 28 distinct field observations as shown in Figure 2. The dependent variable Swaying Level (m/s^2) is summarized in the first column of this matrix, and the three independent variables Flange

Thickness (mm), Operational Speed (km/h), and Wheel Diameter (mm) are summarized in the following columns. The raw data is organized into a clean, numerical format, and reflects the lateral stability of the vehicle as well as the mechanical state of the wheel at each operational event synchronized. The raw data reflects the lateral stability of the vehicle as well as the mechanical state of the wheel at each operational event synchronized, and is therefore organized into a "clean, numerical" format. Finally, this matrix is fed directly into the Microsoft Excel Data Analysis Toolpak for the calculation of descriptive statistics, Pearson correlation coefficients and the final Multiple Linear Regression (MLR) model.

Swaying Level	Flange Thickness	Speed	Wheel Diameter
0.56	32.0	144	850
0.52	32.3	143	848
0.48	31.3	138	847
0.52	30.1	145	846
0.52	31.6	138	844
0.55	32.2	138	842
0.54	29.1	144	841
0.49	32.5	141	840
0.56	31.2	141	839
0.39	30.1	144	838
0.52	29.8	144	837
0.43	32.4	144	832
0.52	29.7	140	831
0.52	30.8	146	830
0.42	31.2	145	829
0.5	32.2	145	826
0.40	32.4	147	825
0.48	29.3	143	823
0.51	31.9	143	819
0.55	31.7	143	818
0.42	32.1	145	817
0.48	28.1	143	815
0.5	32.2	140	812
0.5	30.1	143	810
0.46	32.3	140	809
0.47	31.9	141	808
0.46	29.5	144	802
0.50	31.4	142	792

Fig. 2: Dataset Table of Analysis

3.2 Multiple Linear Regression (MLR) Equation

$$Y=\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3+\epsilon \quad (1)$$

A Multiple Linear Regression (MLR) model was formulated to quantify the combined effect of the parameters. This is a predictive equation where Y is the overall Swaying Level in m/s^2. The y-intercept, $\boldsymbol{\beta}_0$ is the ground for the mathematical relationship as it represents the theoretical swaying level when all other independent variables are set to their exact value of zero. The three main independent variables in the model are: Flange Thickness ($\mathbf{X_1}$) , Operational Speed ($\mathbf{X_2}$), and Wheel Diameter ($\mathbf{X_3}$). These variables are multiplied by the respective regression coefficients, $\boldsymbol{\beta_1}$, $\boldsymbol{\beta_2}$, $\boldsymbol{\beta_3}$ that determine the proportional mathematical influence of the respective parameters. Lastly, there is an error term, ϵ, that represents the unexplained variation in the swaying behavior, or noise, in the equation.

3.3 Mean Squared Error (MSE) Equation

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 \quad (2)$$

For the mathematical validation of the predictive accuracy and internal reliability of this regression model, the Mean Squared Error (MSE) was used. The accuracy of the model is tested with the MSE in an exhaustive manner: the average squared difference between the actual observed values of the swaying Y_i and the model's predictions of the swaying Y ^. In this formula, n is the number of observations in the field and the summation symbol (Σ) is the sum of all the errors in the calculations. The MSE will give more weight to larger prediction errors, since they are squared before being averaged. Therefore, an MSE value close to zero indicates that the predictive model is very close to the real data and there is very little mathematical difference.

4. Results

The results show that the typical wheel degradation and the existing operational speeds have a mathematically weak linear effect on lateral train swaying. The Pearson correlation analysis of the independent variables was used to check for multicollinearity and showed that there was no multicollinearity present, and that operational

speed had the highest correlation with swaying but low (r = -0.330). The resulting Multiple Linear Regression (MLR) model had an extremely small Mean Squared Error (MSE) of 0.0017, confirming the mathematical correctness of the initial evaluation. The model, however, gave low R2 value (R^2 = 0.201) and the individual P-values for flange thickness (0.456), wheel diameter (0.142) and operational speed (0.087) were all greater than the 0.05 significance level. For a vehicle dynamics engineer, this is a very good operating parameter. It mathematically demonstrates Bogie's high intrinsic stability over this operational envelope, ensuring that the system can safely cope with natural wheel wear without causing major instability issues. To this end, the 79.9% of non-explained swaying variance indicates that the dynamics associated with severe lateral hunting are more complex – higher order – than those associated with typical wheel profile degradation.

4.1 Descriptive Statistics

The model was developed using dynamic operational data from 28 field observations, which are then summarized to gain a statistically sound model. Three basic independent variables, Flange Thickness (mm), Operational Speed (km/h), and Wheel Diameter (mm), were used to evaluate the dependent variable, Swaying Level (m/s2). The minimum value of the swaying level for the data points observed was 0.39 m/s^2 and the maximum was 0.56 m/s^2. At the same time, the mechanical parameters were recorded from 850mm to 792mm of wheel diameter to provide a complete window of natural wheel wear and then mathematically isolated to assess their proportionate contribution to the train's swaying behavior. Figure 3 shows the descriptive statistics for the four main variables assessed in the study. This summary represents the range of operation of the vehicle, the central tendency, and dispersion of the swaying level, kinematic speed and mechanical wheel degradation for the maintenance cycles observed.

Variable	Swaying Level (m/s^2)	Flange Thickness (mm)	Speed (km/h)	Wheel Diameter (mm)
Mean	0.4918	31.1214	142.6429	827.5
Median	0.50	31.50	143	829.5
Standard Deviation	0.047	1.2357	2.4223	14.4931
Min	0.39	28.1	138	792
Max	0.56	32.5	147	850

Table.3: Descriptive Statistics

4.2 Pearson Correlation

The Pearson correlation matrix was produced to assess the possible multicollinearity amongst the predictor independent variables and to measure the linear relationship between each other among the independent variables and the dependent variable as in **Figure 4**. From the analysis, it is confirmed that the three independent variables (Flange Thickness, Operational Speed, Wheel Diameter) are not very collinear. There is a very good correlation between the Flange Thickness and Wheel Diameter (r = 0.0660), the value is well below the standard level of concern and is therefore a validation of the structural integrity of the next model, and that is Multiple Linear Regression (MLR). Moreover, the mathematical strength of the correlation coefficients (r) for individual independent parameters and the Swaying Level is poor, with the highest correlation being between Operational Speed and the Swaying Level (r = -0.330). This deep lack of significant linear correlation is in support of the overall conclusion that natural wheel degradation and operational velocities are not significant kinematic factors in causing severe train swaying behavior.

Variable	Swaying Level (m/s2)	Flange Thickness (mm)	Speed (km/h)	Wheel Diameter (mm)
Swaying Level (m/s2)	**1.0000**	**-0.0619**	**-0.3359**	**0.2969**
Flange Thickness (mm)	**-0.0619**	**1.0000**	**-0.1656**	**0.0660**
Speed (km/h)	**-0.3359**	**-0.1656**	**1.0000**	**-0.0849**
Wheel Diameter (mm)	**0.2969**	**0.0660**	**-0.0849**	**1.0000**

Table.4: Pearson Correlation

4.3 Multivariate Linear Regression Modelling

To ascertain the simultaneous influence of wheel degradation and velocity on the swaying level, a multivariate linear regression model was deployed. The mathematical relationship governing this dynamic interaction is defined by the following predictive equation:

$$\text{Swaying Level} = 0.88105 - 0.00531(\textbf{Flange Thickness}) - 0.00645(\textbf{Speed}) + 0.00084(\textbf{Wheel Diameter}) \quad (3)$$

To evaluate the predictive accuracy and internal validity of this equation, an Analysis of Variance (ANOVA) was conducted. The statistical summary of the regression is presented in **Figure 5**. To ensure maximum mathematical precision and eliminate truncation errors caused by high-magnitude variables (such as the 850 mm wheel diameter), the regression coefficients were applied at five decimal places of precision during the Mean Squared Error (MSE) calculation. The exceptionally low MSE of 0.0017 indicates that when the vehicle parameters are strictly calculated, the mathematical deviation of the linear equation is minimal.

Description	Coefficients
Intercept	0.88105
Flange Thickness	-0.00531
Speed	-0.00645
Wheel Diameter	0.00084

Table.5: ANOVA Summary

4.4 The Intrinsic Stability of the Vehicle (Statistical Insignificance)

The most important aspect of this dynamic analysis is the P value of the independent variables. Standard statistical modeling requires a P-value of less than 0.05 to prove a statistically significant relationship. The P-values obtained from this regression model as shown in **Figure 6**, were used to assess the individual effect of the mechanical and kinematic parameters.

Variable	P-Value
Flange Thickness	0.456
Operational Speed	0.087
Wheel Diameter	0.142

Table.6: P-Values

As all parameters are clearly above 0.05, this data clearly shows that any deviation in wheel diameters, flange wear and operational speed in this operational envelope is statistically insignificant in causing trains to sway. Additionally, it is mathematically proven by the R^2 value of 0.201 that the remaining 79.9% swaying behavior not explained by these three variables. This difference in statistics forms a significant engineering insight into the intrinsic dynamic stability of the train. The data shows that the lateral swaying is nearly independent of the natural decrease in the diameter of the wheel and thickness of the flange. In a vehicle dynamics sense, this means that the degradation of the wheel profile that naturally occurs has not affected the equivalent conicity of the wheel-rail contact patch to a critical extent. The conicity is stable and it is not coupled to the car body's suspension system to cause severe resonance for the bogie. Furthermore, although operational speed shows a weak influence (thus

approaching the significance threshold of 0.087), it does not exceed it.

4.5 Unexplained Variance and Higher-Order Dynamic Interactions

A scatter plot of the multivariate regression fit is shown in **Figure 3** to visually validate the above statistical results. The plot compares the actual swaying levels that are measured in the field with the swaying levels predicted by the model. A good prediction would have data points that would line up in a straight diagonal line that would have little error. But the data points in **Figure 3** do show a distinctly horizontal dispersion. Actual field measurements vary greatly over the operating range, but the predictions of the model are mathematically limited to a very narrow range. This horizontal grouping is meant to serve as a visual affirmation that such differences in basic wheel wear and operating speed have not the necessary impact to introduce into the vehicle undue lateral instability.

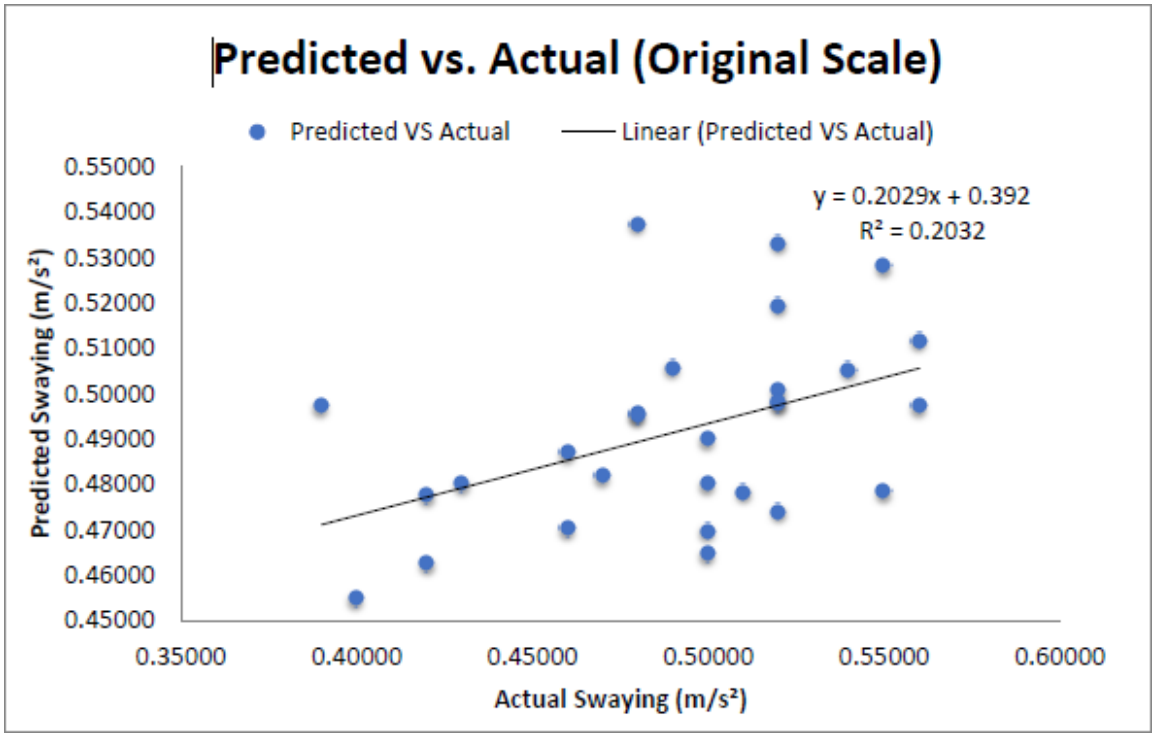


Fig.3: Predicted vs. Actual Swaying Level (Multivariate Regression Fit)

5.Conclusion

The statistical analysis demonstrates that natural wheel degradation and current operational velocities do not possess a significant linear correlation with severe train swaying. The Multiple Linear Regression (MLR) model yielded P-values for flange thickness (0.456), wheel diameter (0.142), and operational speed (0.087), all exceeding the 0.05 significance threshold. From a vehicle dynamics perspective, this confirms that natural profile wear does not critically alter the equivalent conicity of the wheel-rail contact patch, and current speeds (138–147 km/h) remain strictly within safe kinematic limits, well below the critical hunting speed. Furthermore, the model's coefficient of determination (R^2 = 0.201) leaves 79.9% of the swaying variance unexplained. By mathematically excluding standard wheel wear and speed as primary drivers, this study validates the strong intrinsic mechanical stability of the bogie system. Consequently, future swaying mitigation strategies must pivot away from standard wheel reprofiling towards investigating higher-order complex dynamics, specifically external track irregularities and secondary suspension system degradation.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The author confirm contribution to the paper as follows: **Multivariate Analysis (MA) of Train Swaying Dynamics Based on Wheel Parameters and Operational Speed To Validate Overall Train Swaying Behavior**: Nagheswaran A/L Srinivasan, **Supervisor**: Prof. Madya Ir. Ts. Dr. Musli Nizam bin Yahya and Ir. Ts. Norazman Bin Abu Hassan. All authors reviewed the results and approved the final version of the manuscript.

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